Seminar: Introduction to Cointegration
Applied Econometrics

Jozef Barunik

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Summer Semester 2009/2010
Outline of the today’s talk

- Cointegration: definition and some intuition
- Testing cointegration: simple test and the Engle-Granger procedure
- Example: Money demand equation
- Error Correction Model (so called ECM)
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Cointegration - Intuition

Shock to "Blue" caused only temporary shift from the equilibrium relation; corr = 0.941

Shock to "Blue" caused shift in equilibrium; corr = 0.923
Definition of Cointegration

- Formal definition: An \( (n \times 1) \) vector time series \( x_t \) is said to be cointegrated if each of the series taken individually is \( I(1) \), that is, nonstationary with a unit root or integrated of order 1, while some linear combination \( \beta'x_t \) is stationary, or \( I(0) \), for some non-zero \( (n \times 1) \) vector \( \beta \).

- Long-term stable relationship between two (or among many) variables: something like equilibrium among those variables exists.

- Those variables cannot wander off in a long term, they must arrive to its equilibrium level.

- Equilibrium: \( \beta_1x_{1,t} + \beta_2x_{2,t} + \cdots + \beta_nx_{n,t} = 0 \)

- The equilibrium error: \( e_t = \beta'x_t \sim \text{stationary.} \)
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Cointegration and Correlation

- Cointegration and Correlation: two things about the same?
  - Correlation: if one variable moves up, the second will do the same.
  - Cointegration: in case of shock in one variable, their long-term relationship would not change.

Note:

Cointegrating relationships are unusual and very important as they give the information about the long-term behavior.
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Testing Cointegration

- Natural approach to test cointegration:
  - Take the difference of two $I(1)$ series and the result should be stationary:

\[ y_t = \alpha + \beta x_t + u_t \Rightarrow y_t - \beta x_t - \alpha = u_t \]  

- But: $\beta$ superconsistent and OLS designed to produce stationary residuals. Thus slightly different critical values that are more strict about the properties of $u_t$.

- Another reason for different crit. values: coefficients $\beta$ are estimated, they are not true values (this holds only asymptotically)
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Example: Money Demand

- Relation among money and real economy:

**money demand equation**

\[ m_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t + \epsilon_t \]

- Demand for money:
  - Individuals want to hold a real quantity of money balances (real proportion of nominal money given by price level).
  - The transaction demand: real money demand depends on amount of goods that is intended to be bought.
  - Speculative motive: interest rate represents opportunity costs of cash money.

- Solving for \( \epsilon_t : \epsilon_t = m_t - \beta_0 - \beta_1 p_t - \beta_2 y_t - \beta_3 r_t \)

- Linear combination of \( m_t, y_t, r_t, p_t \) should be stationary thus these variables should be cointegrated if the money demand equation holds.
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Engle-Granger Procedure and Money Demand

1. Test the order of integration for all variables by ADF
2. Estimate (by OLS) \( X_t = \alpha_0 + \beta_0 Y_t + u_t \), where \( Y_t \) is vector of variables
3. Test \( u_t \) for the presence of unit-root.

Load the data

Gretl sample file: Greene ⇒ Greene 5.1 U.S. Macro data
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Logs of all variables, plot each of them

Gretl Code

logs realgdp cpi_u M1
scatters tbilrate l_realgdp l_cpi_u l_M1
Engle-Granger Procedure and Money Demand

- Estimate $m_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t + \epsilon_t$ and save the residuals

**Gretl Code**

```gretl
ols l_M1 const l_realgdp tbilrate l_cpi_u
genr uhat3 = $uhat
```

- Test residuals for unit-root (without const)

**Gretl Code**

```gretl
adf 4 uhat3 - - c - - verbose
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Engle-Granger Procedure and Money Demand

- Now use the Engle-Granger procedure

Gretl Code

```gretl
coint 4 l_M1 l_realgdp l_cpi_u tbilrate
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- Any difference?
- Cointegration rejected

Note

Alternatively, use Gretl Menu to do the Engle-Granger procedure:
Model → Time series → Cointegration test → Engle-Granger
Engle-Granger Procedure and Money Demand

- Now use the Engle-Granger procedure

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Results: Engle-Granger Procedure and Money Demand

- What might help?
  - Dependent variable M-P (realmoney)
  - More observations

Gretl Code

genr realmoney=l_M1-l_cpi_u
gnuplot realmoney - - with-lines - - time-series
ols realmoney const l_realgdp

coint 4 realmoney l_realgdp tbilrate

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Complete Engle-Granger Procedure

- Test the order of integration for all variables by ADF
- Estimate (by OLS) \( X_t = \alpha_0 + \beta_0 Y_t + u_t \), where \( Y_t \) is vector of variables
- Estimate the error-correction model \( \Delta X_t = \alpha_0 + \beta \Delta Y_t + \rho u_{t-1} + \epsilon_t \), (sometimes lags of \( \Delta X_t \) and \( \Delta Y_t \) needed; \( u_{t-1} \) comes from the step 2)
- Evaluate the model adequacy (the parameter \( \rho \) is expected to be negative and can be interpreted as the speed of adjustment as the \( u_{t-1} \) is the error correction term.)

Gretl Code

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ols realmoney const l_realgdp l_cpi_u l_M1
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Jozef Barunik (IES, FSV, UK)  
Seminar: Introduction to Cointegration  
Summer Semester 2009/2010
Complete Engle-Granger Procedure

Dependent variable: d_realmoney

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.00160546</td>
<td>0.00105943</td>
<td>1.514</td>
</tr>
<tr>
<td>whst10_1</td>
<td>-0.0393710</td>
<td>0.0156320</td>
<td>-2.519</td>
</tr>
<tr>
<td>d_cbi15te1</td>
<td>-0.00113736</td>
<td>0.00124243</td>
<td>-0.902</td>
</tr>
<tr>
<td>d_l_realgd1p</td>
<td>-0.0787934</td>
<td>0.0875293</td>
<td>-0.092</td>
</tr>
<tr>
<td>d_realmoney_1</td>
<td>0.420809</td>
<td>0.191196</td>
<td>2.163</td>
</tr>
</tbody>
</table>

Mean dependent var 0.001427 S.D. dependent var 0.013458
Sum squared resid 0.023546 S.E. of regression 0.011062
R-squared 0.946624 Adjusted R-squared 0.939317
F(4, 197) 25.89777 P-value(F) 2.94e-17
Log-likelihood -626.8613 Akaike criterion -1253.726
Schwarz criterion -1227.181 Hannan-Quinn -1237.030
rho -0.149895 Durbin’s h -4.776732
Example: ECM and Simulated Data

...load dataAE7simulated.xls
Example: ECM and Simulated Data

Model 3: OLS, using observations 3-50 (T = 48)
Dependent variable: d_zz

<table>
<thead>
<tr>
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<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.275286</td>
<td>0.219538</td>
<td>1.254</td>
<td>0.2163</td>
</tr>
<tr>
<td>d_yy_1</td>
<td>-0.161983</td>
<td>0.126099</td>
<td>-1.285</td>
<td>0.2055</td>
</tr>
<tr>
<td>uhat1_1</td>
<td>-0.634589</td>
<td>0.184766</td>
<td>-3.435</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

Mean dependent var 0.207359  S.D. dependent var 1.669873
Sum squared resid 101.8840  S.E. of regression 1.594699
R-squared 0.213211  Adjusted R-squared 0.178242
F(2, 45) 6.097235  P-value(F) 0.004537
Log-likelihood -86.17226  Akaike criterion 178.3445
Schwarz criterion 185.9581  Hannan-Quinn 180.4659
rho -0.067311  Durbin-Watson 2.103659

![Actual and fitted d_zz](image)
Cointegration is a strong, long-term relationship among variables. It occurs if all share a common trend or if there is some form of equilibrium relationship as in money demand equation. It implies much stronger codependence than correlation. To test cointegration, Engle-Granger procedure is used. Cointegration implies presence of error-correction mechanism.
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Engle-Granger Procedure - More Examples

- Danish money demand: Gretl sample files  gretl  denmark
  - Contains data about real money balances, real income, interest rates on bonds and deposits. If only bond rate of these two used, cointegration confirmed by Engle-Granger procedure although on much smaller sample. The plot of residuals follows.
- Let’s estimate...
Engle-Granger Procedure - More Examples

- Danish money demand: Gretl sample files gretl denmark
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Questions

Thank you for your Attention!