Fast Method of Sparse Acquisition and Reconstruction of View and Illumination Dependent Datasets

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Abstract

Although computer graphics uses measured view and illumination dependent data to achieve realistic digital reproduction of realworld material properties, the extent of their utilization is currently limited by a complicated acquisition process. Due to the high dimensionality of such data, the acquisition process is demanding on time and resources. Proposed is a method of approximate reconstruction of the data from a very sparse dataset, obtained quickly using inexpensive hardware. This method does not impose any restrictions on input datasets and can handle anisotropic, non-reciprocal view and illumination direction-dependent data. The method's performance was tested on a number of isotropic and anisotropic apparent BRDFs, and the results were encouraging. The method performs better than the uniform sampling of a comparable sample count and has three main benefits: the sparse data acquisition can be done quickly using inexpensive hardware, the measured material does not need to be extracted or removed from its environment, and the entire process of data reconstruction from the sparse samples is quick and reliable. Finally, the ease of sparse dataset acquisition was verified in measurement experiments with three materials, using a simple setup of a consumer camera and a single LED light. The proposed method has also shown promising performance when applied to sparse measurement and reconstruction of BTFs, mainly for samples with a lower surface height variation. Our approach demonstrates solid performance across a wide range of view and illumination dependent datasets, therefore creating a new opportunity for development of time and cost-effective portable acquisition setups.

Keywords: apparent BRDF, measurement, reconstruction, sparse sampling, portable setup, BTF

1 1. Introduction

View and illumination dependent data can be beneficial in 2 3 many computer graphic applications, due to their ability to dig-4 itally represent the actual appearance of respective material. 5 However, their measurement is costly and time-consuming, be-6 cause standard acquisition procedures of such data often require 7 lengthy measurements, or either a specific shape of the mea-8 sured sample or a dedicated measurement setup. Bidirectional ⁹ reflectance distribution function (BRDF) [25], spatially varying ¹⁰ BRDF (SV- BRDF) and bidirectional texture function (BTF) [3] 11 are examples of such data. While a four-dimensional BRDF de-12 scribes distribution of energy reflected to the viewing direction ¹³ when illuminated from a specific direction, a six-dimensional 14 SVBRDF additionally captures the spatial dependency of re-15 flectance across a material surface. While BRDF and SVBRDF 16 impose restrictions on reciprocity, opacity and a range of sam-17 ple height variations, the six-dimensional BTF generally does 18 not fulfill these restrictions. This is due to local effects in a ¹⁹ rough material structure such as occlusions, masking, subsur-20 face scattering, and inter-reflections.

²¹ Therefore, individual BTF pixels are not regarded as BRDF ²² but rather apparent BRDF (ABRDF). If we process individ-



Figure 1: Parameterization (left) of view and illumination-dependent data of a single/average pixel (right).

²³ ual color/spectral channels separately, the ABRDF can be rep-²⁴ resented by a four-dimensional function *ABRDF*($\theta_i, \varphi_i, \theta_v, \varphi_v$). ²⁵ ABRDF is the most general data representation of a reflectance ²⁶ of opaque materials dependent on local illumination $I(\theta_i, \varphi_i)$ ²⁷ and view $V(\theta_v, \varphi_v)$ directions; therefore, we focus on its proper ²⁸ acquisition and reconstruction in this paper. Its typical param-²⁹ eterization by elevation θ and azimuthal φ angles is shown in ³⁰ Fig. 1-left. A projection of the 4D ABRDF, representing de-³¹ pendence of view and illumination directions of a single pixel ²⁸ (BTF) or its average value (BRDF) by means of a 2D image, is ³⁹ shown in Fig. 1-right. Note that individual rectangles (an exam-³⁴ ple is shown in red) represent 2D subspaces of 4D ABRDF at ³⁵ constant elevations (θ_i/θ_v). These subspaces are toroidal. That ³⁶ is data of the highest $\varphi \approx 2\pi$ are followed by data of the lowest ³⁷ $\varphi \approx 0$.

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Main contributions of this paper:

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- a reconstruction method of the entire anisotropic ABRDF space from less than two hundred sparsely measured samples
- a practically verified novel method for intuitive and fast ABRDF acquisition and reconstruction using a consumer camera and LED light in under 10 minutes.

Main features of the proposed method:

- a correct reconstruction of non-reciprocal, energy nonconserving ABRDF data
- an arbitrarily dense sampling of specular highlights, without increasing measurement time
- no need for lengthy measurement using a dedicated and expensive measurement setup
 - not necessary to process or extract the measured sample from its environment
 - contrary to analytical BRDF models, this method requires neither a lengthy fitting procedure nor guessing at initialization values.

This paper is structured as follows: Section 2 sets work in the context of related research. Section 3 explains the principle of the proposed method. Section 4 shows results of performed experiments. The method's limitations are discussed in Section 5, and pilot project results of the real data acquisition scenario are shown in Section 6. Section 7 shows experimental reconstruction of BTF samples, and Section 8 concludes the paper.

48 2. Prior Work

The proposed work relates to methods of BRDF or SVBRDF
 acquisition and interpolation from sparse samples.

Such data were initially captured by setups based on go-51 52 nioreflectometers realizing a required four mechanical degrees 53 of freedom (DOF) of camera/light/sample movement [12], [29]. 54 Because measurement times were too long, setups were used 55 which reduced the required number of DOF using parabolic ⁵⁶ mirrors [4], or kaleidoscope [10]. They allowed the capture 57 of many viewing directions simultaneously; however a limited 58 range of surface height or elevation angles resulted. Measure-⁵⁹ ment time can also be reduced by using multiple lights and sen-60 sors simultaneously [20]; yet, high financial cost is associated 61 with such a setup. Another group of fast acquisition methods 62 reduces the number of DOF by using a known shape of the 63 sample [19], [32], [17], [23], [13]. However, these approaches 64 are often limited to isotropic BRDFs, or focus on represent-65 ing sparsely sampled data using a parametric BRDF model. 66 There is an existing statistical acquisition approach [22] allow-67 ing quick and economical measurement of ABRDF; however, it 68 requires several samples of material with regular structure, po-69 sitioned in different orientations with respect to the camera. Un-⁷⁰ fortunately, methods [22],[23] require a specific sample shape 71 or placement coupled with its extraction from the original envi-72 ronment. Isotropic SVBRDF can also be estimated from pho-73 tometric stereo using a parametric reflectance model [9] or bi-⁷⁴ variate BRDF [1]. Finally, it is possible to use portable setups

⁷⁵ measuring SVBRDF by matching sparsely locally measured
⁷⁶ isotropic BRDFs (using condenser lens optics) with sparsely
⁷⁷ measured global reflectance fields [6]. Another approach records
⁷⁸ SVBRDF from a single view using 1 DOF-moving linear light
⁷⁹ source and a set of known BRDF samples recorded simultane⁸⁰ ously with the sample [27]. Recently, sparse SVBRDF mea⁸¹ surement and reconstruction have been used based on the mea⁸² surement of several images of known geometry illuminated by
⁸³ a circularly polarized light [8]. Although this method requires
⁸⁴ the capture of only four sample images, its usage is limited to
⁸⁵ flat and isotropic measurements and it requires a complex mea-

View and illumination dependent data interpolation is often performed when sparse images of known geometry and ilulmination direction are recorded. Reflectance data collected of from such images can be interpolated either by a parametric reflectance model [17], or in the form of isotropic BRDFs interpolated by means of three-dimensional radial basis functions [32] in achieving a reconstruction of SVBRDF. Alternatively, a 4 4D BRDF can be decomposed into simpler 1D and 2D composonents having physical meaning, to allow parametric editing of visual properties [16].

⁹⁷ However, to the best of our knowledge, no measurement ⁹⁸ technique yet exists enabling rapid capture of anisotropic ABRDF ⁹⁹ using an consumer camera and light. Proposed is a method ¹⁰⁰ for fast, non-restricted anisotropic ABRDF space reconstruc-¹⁰¹ tion from extremely sparse samples that can be measured in a ¹⁰² few seconds by continual movement of the camera and light. ¹⁰³ Contrary to parametric BRDF models [24] or other simplified ¹⁰⁴ solutions (see Fig. 2), this method is capable of correctly recon-¹⁰⁵ structing non-reciprocal, non-energy-conserving ABRDF data.



Figure 2: A comparison of renderings based on a single texture modulated by an analytical BRDF model [14] (left), reference BTF measurements using 6561 images (middle), and the proposed sparse data selection and reconstruction using 168 images (right).

¹⁰⁷ Although the principle of the method has been outlined in ¹⁰⁸ [7], this paper provides additional thorough reasoning of the ¹⁰⁹ method's functionality. In addition, an introduction to a novel ¹¹⁰ interpolation technique for missing elevations, results of real ¹¹¹ appearance acquisition, and the method's application on sparse ¹¹² reconstruction of BTF datasets are included.

113 3. The Proposed Reconstruction Method

A robust and sparse acquisition of general view and illu-¹¹⁵ mination dependent appearances is a tricky task. While a po-¹¹⁶ sition of specular highlights can be expected near the mirror ¹¹⁷ reflection, the location of anisotropic highlights is unknown. It 119 geometry of the measured surface. In our work we look for a 164 anisotropic properties (mutual positions of the light and cam-¹²⁰ very sparse set of illumination/view measurement points. They 121 should allow a visually tolerable reconstruction of material re-122 flectance, as well as a quick measurement of the sparse dataset 167 ¹²³ using simple inexpensive hardware. There would be no need to ¹⁶⁶ *slice* s_D (blue), i.e., $\varphi_i + \varphi_v = 2\pi$ holds for azimuthal angles. The 124 preprocess the measured material sample or remove it from its 125 environment.

Standard angularly uniform or even adaptive sampling strate- 171 ple (see Fig. 14-right)). 127 gies require many samples to preserve high frequencies in the 128 data. On the other hand, employing analytical BRDF models 129 imposes restrictions on data reciprocity and requires lengthy 130 fitting, etc. Therefore, we analyzed a typical ABRDF and em-¹³¹ ployed this knowledge to capture and reconstruct its behavior 132 using a small set of measurements. This analysis has shown that 133 it is most effective to place samples perpendicularly to specular 134 highlights in a subspace of view/illumination azimuths. This 135 is done in such a way that the samples form slices in the sub-136 space and can be easily measured by horizontal movement of 137 the light/camera around the measured sample. As the appear-138 ance of the azimuthal subspaces often depends on elevation an-139 gles, to create a more precise approximation we suggest sam-140 pling four combinations of view/illumination elevations.

A principle of the proposed method [7] is explained in Fig. 3. 141 First, the material's ABRDF (Fig. 3-a) is sparsely measured in



Figure 3: Example of ABRDF reconstruction: (a) original, (b) sparse-sampling using 8 slices, (c) reconstructions of elevations where the slices were measured, (d) missing data interpolation.

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143 four subspaces by means of eight slices (Fig. 3-b), then the 144 missing values in these subspaces are reconstructed from the 145 values of the slices (Fig. 3-c), and lastly the remaining values at 146 non-measured elevations are interpolated (Fig. 3-d).

147 3.1. Acquisition of Slices

Because one of our major concerns is the simplicity and 148 149 speed of the acquisition process, we suggest taking samples by 150 the continuous movement of light/camera around the sample at 151 fixed elevations. By doing so, samples can be taken at an ar-152 bitrary density, limited only by camera movement speed and ¹⁵³ frame-rate. Each subspace of azimuthal angles φ_i/φ_v is sam-154 pled by means of two perpendicular slices (see Fig. 5-a), which 155 differ in the direction of mutual movement of camera and light. ¹⁵⁶ In principle, the slices are, for a majority of the materials, or-157 thogonal to their most prominent features: a specular reflection 158 and an anisotropic reflection (see Fig. 4). These features are of-159 ten constant in the direction perpendicular to the slices and thus 160 can be effectively represented by their marginal values.

The slice aligned with the direction of the specular high-¹⁶² lights is called *axial slice* s_A (red), i.e., $\varphi_v - \varphi_i = \alpha$ holds

118 depends on the local macro-geometry as well as on the micro- 163 for azimuthal angles. The axial slice records the material's ¹⁶⁵ era are fixed while the sample rotates (see Fig. 14-left)), i.e., 166 its value is almost a constant for near-isotropic samples.

> The slice perpendicular to the highlights is called *diagonal* 169 diagonal slice captures the shape of the specular peaks (light 170 and camera travel in mutually opposite directions over the sam-



Figure 4: Examples of ABRDF toroidal subspaces for derivation of optimal placement of the axial slices (green and red alternatives).

We focused first on analysis of the ABRDF subspace hav-173 ing the highest elevations, in which the illumination and view 174 dependent effects are the most pronounced. See in the first row 175 of Fig. 7. The azimuthal difference of light and camera during 176 axial slice measurement – α influences the placement of slices 177 in the ABRDF toroidal subspace; therefore, we analyzed opti-178 mal placement of axial and diagonal slices across a number of 179 ABRDFs. The study has shown that while the placement of a 180 diagonal slice can be arbitrary, the highest variance along axial 181 slices is achieved near the specular highlight. Consequently, 182 this is - most likely - the best placement of the axial slice $_{183} \alpha = 180^{\circ}$ (green dots in Fig. 4). However, such a placement 184 might omit vital color/luminance information in some parts of 185 the subspace. For example, it would completely miss yellow 186 anisotropic features as in Fig. 4-a or dark parts as in Fig. 4-b. 187 Therefore, we used the slice with the second highest variance $_{188} \alpha = 15^{\circ}$ (red dots in Fig. 4) to eliminate occlusion of the camera 189 with the light and capture most visual features of the ABRDF 190 subspace (see the shift of the red axial slice from image diago-191 nal in Fig. 5-a).

For experimental purposes the slices can be taken from the 192 193 measured ABRDF (Fig. 5-a) as

$$s_{A,\theta_i\theta_\nu}(\varphi_i) = ABRDF(\theta_i, \theta_\nu, \varphi_i, \varphi_\nu = \varphi_i + \alpha) , \qquad (1)$$

$$s_{D,\theta,\theta_\nu}(\varphi_\nu) = ABRDF(\theta_i, \theta_\nu, \varphi_i = 2\pi - \varphi_\nu, \varphi_\nu) .$$

194 3.2. Reconstruction from slices

ABRDF toroidal subspace reconstruction is performed for 196 elevation angles at which the slices were captured. It can be 197 explained as a combination of two slices (i.e., sets of marginal ¹⁹⁸ values) as shown in Fig. 5. The reconstruction of point ¹⁹⁹ $ABRDF(\theta_i, \theta_v, \varphi_i, \varphi_v)$ in ABRDF subspace starts with combin- $_{200}$ ing contributions of the s_A and s_D slices. We tested their sum 201 and product; however, the latter improperly enhanced the loca-202 tions at intersections of the specular and anisotropic highlights 203 as shown in Fig. 6-b. Note that the sum of slice contributions 204 (see Fig. 6-c) preserves specular highlights, which are less af-205 fected by the anisotropic highlights. Therefore, we finally used 206 the sum of slices in our reconstruction procedure:

$$y_{\theta_i\theta_v}(\varphi_i,\varphi_v) = s_{A,\theta_i\theta_v}(\varphi_{i,R}) + s_{D,\theta_i\theta_v}(\varphi_{v,R}) , \qquad (2)$$



Figure 5: Reconstruction of a toroidal ABRDF subset from two slices at fixed elevations: (a) reference data with slice placements, (b) data profiles in the slices, (c) reconstruction from slices ($\frac{\pi}{4}$ rotated), (d) final reconstruction.



Figure 6: Original subspace (a), and its reconstructions using product of slices (b) and sum of slices (c). Below are the difference values in: CIE Δ E / RMSE / PSNR[dB].

Note that the original azimuths φ_i, φ_v had to be rotated for $\pi/4$ (Fig. 5-c) to account for the slant of slices with respect to φ_i, φ_v coordinate system (Fig. 5-a). Finally, the summed $\pi/4$ value v is mapped to a dynamic range of original slices

$$\widehat{ABRDF}(\theta_i, \theta_v, \varphi_i, \varphi_v) = v_{\theta_i \theta_v}(\varphi_i, \varphi_v) \cdot (M - m) + m \quad (3)$$

$$m = \min(s_{A,\theta_i\theta_\nu} \cup s_{D,\theta_i\theta_\nu}) \qquad M = \max(s_{A,\theta_i\theta_\nu} \cup s_{D,\theta_i\theta_\nu}) \quad .(4)$$

Since the axial slice always has a constant value for isotropic and samples, the slices do not have to be combined and reconstruction can be performed using the diagonal slice alone as

²¹⁶
$$ABRDF(\theta_i, \theta_\nu, \varphi_i, \varphi_\nu) = s_{D,\theta_i\theta_\nu}(\varphi_{\nu,R})$$
 (5)

Fig. 7 shows a reconstruction of anisotropic ABRDF sub-218 space at elevations $\theta_i/\theta_v = 75^o/75^o$ (the second row) from two 219 slices (the third row) and prove the ability of the proposed approach to represent a variaty of anisotropic materials

proach to represent a variety of anisotropic materials.



Figure 7: Comparison of the material's ABRDF toroidal subspace at elevation 75^{o} (the first row), with its reconstruction (the second row) from the axial (red) 220 and diagonal (blue) slices (the third row).

221 3.3. Interpolation of missing values

At this point, sparse acquisition and reconstruction of the ABRDF subspace has been explained. However, the selection

²²⁴ of elevations at which the slices are measured significantly in-²²⁵ fluences the final ABRDF reconstruction. Therefore, we per-²²⁶ formed an experiment with two measured ABRDFs (isotropic ²²⁷ specular and diffuse anisotropic material) in order to find the ²²⁸ proper combination of two elevations at which the four sub-²²⁹ spaces should be measured. We tested six different combina-²³⁰ tions of elevations. Only samples from these illumination/view ²³¹ elevations were used for the entire ABRDF interpolation us-²³² ing radial basis functions [26]. The average RMSE differences ²³³ between ground-truth ABRDF data and interpolation shown in ²³⁴ Fig. 8 suggest that the combination of $\theta = 45^{o}/75^{o}$ provides the lowest reconstruction error. Therefore, we have chosen



Figure 8: ABRDF reconstruction error (RMSE) for different elevation combinations used for selection of four measured subspaces.

²³⁶ the highest elevation angles $\theta_i = \theta_v = 75^\circ$, where the specu-²³⁷ lar reflections are the most intensive (see first row of Fig. 12). ²³⁸ The lower elevation angles were decreased to $\theta_i = \theta_v = 30^\circ$ ²³⁹ (the second best choice from Fig. 8) for better representation ²⁴⁰ of material appearance at orthogonal viewing and illumination ²⁴¹ directions, which are the most visually salient. More than these ²⁴² four subspaces can be used at the expense of more camera/light ²⁴³ elevations; however, this would increase the number of sam-²⁴⁴ ples and the complexity of their measurement. Finally the four ²⁴⁵ subspaces at the following elevations were sampled: $\theta_i/\theta_v =$ ²⁴⁶ $30^\circ/30^\circ, 75^\circ/75^\circ, 30^\circ/75^\circ, 75^\circ/30^\circ,$).

However, data for the remaining subspaces are still unknown and have to be estimated. The BRDF parametric models, e.g., [23], cannot be used to solve this problem because they impose restrictions on data properties (reciprocity, energy conservation, estimated), require many more samples or a different distribution of samples, and lengthy fitting. They also depend on initial values. We tried to fit measured samples using the anisotropic paramettric BRDF model [14]. Due to a low number of samples and their distribution we were unable to find a stable parameter fit for most of the tested ABRDFs. Moreover, these models are not designed to handle non-reciprocal ABRDF data. Therefore, we tested the following two interpolation approaches:

Method A – In the first one, the interpolation was performed by means of the four-dimensional radial basis functions [26] computed separately in each color channel. We tested several parameterizations of illumination and viewing directions, e.g., $[\theta_i, \varphi_i, \theta_v, \varphi_v], [\theta_h, \varphi_h, \theta_d, \varphi_d]$ from [28], and finally used parameterization according to [11], applied to both illumination and view directions $[\alpha_i, \beta_i, \alpha_v, \beta_v]$. This parameterization has shown the lowest reconstruction error due to alignment of specular highlights 0° value of angle β_i .

²⁶⁸ **Method B** – Due to relatively high computational demands of ²⁶⁹ Method A, we developed a faster hybrid linear interpolation ²⁷⁰ constrained by a reflectance model. This interpolation consists 271 of two steps shown in Fig. 9. First we interpolate data at dif- 309 272 ferent viewing and constant illumination elevations. Then the 310 proaches provide similar visual quality (see more discussion remaining illumination elevations are filled. In each interpo-



Figure 9: Steps of the interpolation approach A.

274 lation step, the average ABRDF value for each unknown ele-275 vation was approximated by fitting a simplified monospectral ²⁷⁶ one-lobe Lafortune model [15] with parameters k,α to known 277 slice values

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$$f_r(k,\alpha) = k(\cos\theta_i \cdot \cos\theta_v)^{\alpha} \quad . \tag{6}$$

²⁷⁹ Initialization of k, α was constant during all experiments. Val- $_{280}$ ues obtained from the model at elevations θ are scaled by the 281 mean values of the slices and then used for obtaining inter-282 polation weights. These weights are then applied for a linear 283 interpolation of missing elevations from slice values at known $_{284}$ elevations as shown in Fig. 10. Elevations lower than 30° are 285 extrapolated using the scaled model's (6) predictions. This pro-286 cedure is performed over all azimuthal directions as shown in Fig. 9.



Figure 10: An interpolation of non-measured elevation values.

Method A is approximately three times more computation-288 289 ally intensive than Method B and provides better results in most ²⁹⁰ cases. While Method A allows arbitrarily dense sampling, even for originally unmeasured azimuthal directions, Method B re-291 ²⁹² constructs ABRDF data in their original azimuthal sampling. The results in Fig. 11 show the major visual differences be-²⁹⁴ tween both proposed interpolation methods in two illumination 295 environments.

We also tested modification of this step-wise interpolation 29 297 of subspaces (Fig. 9) using a displacement interpolation (de-²⁹⁸ noted as BD) method [2]. Compared to the weighted linear ²⁹⁹ interpolation (Method B), its principle is based on solving the 300 generalized mass transport optimization problem. As this method 321 this paper was 81 view ×81 illumination directions (6561 val-301 cannot extrapolate, the low elevation areas were reconstructed 302 using Method B. Method BD gives better results than Method B and comparable results to Method A. The whole subspace in-303 ³⁰⁴ terpolation is about thirty times slower than Method A, while when view or illumination direction is fixed, i.e., individual 306 lines in subspaces are interpolated separately, its speed com-307 pares to Method A. For this reason we have not used Method 308 BD further in the paper.

Although both global (A) and local (B) interpolation ap-³¹¹ in Section 4), we believe their performance can be further im-312 proved, e.g., using estimated height-map as an interpolation 313 constraint.



Figure 11: A comparison of the interpolation methods performance in grace and st.peters illumination environments [5]. Below are the CIE $\Delta E / PSNR[dB] / SSIM / VDP2$ difference values.

314 4. Results of Simulated Measurement

315 In this section we show results of sparse reconstruction ex-316 periments performed on isotropic BRDF and anisotropic ABRDF 317 data. The data served as a source of sparse sampling and were 318 simultaneously used for evaluating reconstruction quality of the 319 method.

Generally, the angular resolution used in all experiments in 320 ³²² ues) [29] distributed uniformly over the hemisphere (Fig. 1) To 323 sample this resolution in the slices of the proposed method we 324 need only 168 samples to obtain information sufficient for data 325 reconstruction.

326 In the first experiment, we tested our method on reconstruc- $_{327}$ tion of 55 isotropic BRDF samples (resampled to 81×81 di-³²⁸ rections) from the MERL BRDF database [19]. The advantage $_{329}$ of isotropic reconstruction is that only four diagonal slices s_D ³³⁰ have to be obtained (in our case 84 samples instead of the 168 ³⁸³ 5. Limitations ³³¹ needed for anisotropic data). Mean reconstruction errors of all $_{332}$ 55 BRDFs (8bits/channel) were: CIE $\Delta E=9.1$, RMSE=15.7, ³³³ and PSNR= 24.9 [7].

In the second experiment, ten BTF samples (nine from Bonn 334 ³³⁵ University BTF database¹ and one from Volumetric Surface ³³⁶ Texture Database²) were used (aluminum profile, corduroy, dark 337 and light fabrics, dark and light leatherettes, lacquered wood, knitted wool, upholstery fabric Proposte, and Lego). These ma-338 terials, due to their rough structure and often non-opaque prop-340 erties, exhibit anisotropic effects of occlusions, masking, sub-341 surface scattering and therefore represent a challenging dataset to test the proposed method. All BTF pixels were averaged to ³⁴³ obtain the average ABRDF of the material (first row of Fig. 12). The results of the complete reconstruction of original ABRDFs 344 345 from 168 sparse samples are shown in Fig. 12. Together with 346 difference images (10× scaled) and reconstruction errors in terms $_{347}$ of CIE Δ E, RMSE, PSNR, these results show that even a very 348 sparse set of measured values can provide promising recon-³⁴⁹ struction of such challenging anisotropic datasets. Although on ³⁵⁰ average both interpolation approaches performed similarly, the ³⁵¹ difference images in Fig. 12 show that the global interpolation 352 Method (A) estimated incorrect values for elevations between $_{353}$ the two sampled elevation values (30° and 75°). On the other ³⁵⁴ hand, Method (B) gives, due to a lack of global knowledge, the worst estimation for extrapolated elevations. That is, elevations $_{356}$ smaller than 30° as represented by the first few rows/columns 357 in the images. Finally, the displacement interpolation – Method 358 BD scored similarly to Method A. The reconstruction and inter-₃₅₉ polation of a single ABRDF from 168 samples take \approx 1 second $_{360}$ using interpolation Method A, ≈ 0.3 second using Method B, $_{361}$ and ≈ 1 second using Method BD on Intel Xeon 2.7GHz (using 362 3 cores).

To validate the contribution of our method, we compared 363 ³⁶⁴ its reconstruction performance using 168 samples with the uni-365 form sampling of a similar samples count. For this purpose, 366 hemispheres of illumination and viewing directions were sam-³⁶⁷ pled by means of 13×13 samples, producing a total of 169 sam-368 ples. Then the missing values in the ABRDF space were inter-³⁶⁹ polated from these sparse samples by means of four-dimensional radial basis functions [26] (Method A). The interpolation was ₃₇₁ computed separately in each color channel, and $0 \approx 2\pi$ discon-372 tinuity has been avoided using the onion parameterization of 373 illumination and view directions [11]. Comparison of ten interpolated ABRDFs (see Fig. 13) has shown that the proposed reconstruction method has a better performance than the interpo-375 lation from uniform samples, mainly near specular highlights, 376 377 as confirmed by the objective criterion values shown below the reconstructions. On average, the proposed reconstruction provides 1.4 and 3.2 lower $\Delta E / RMSE$ values and 1.8 higher PSNR 380 value across ten tested ABRDFs. Moreover, the data acquisi-381 tion process using our method is considerably faster and less ³⁸² demanding on hardware as shown in Section 6.

¹http://btf.cs.uni-bonn.de/

The limitations of the proposed method are threefold. First, 385 since the method restores reflectance at given elevations only 386 from two orthogonal slices, it cannot reliably capture features 387 that are not orthogonal to the slices (see second example of ³⁸⁸ corduroy in Fig. 7). It must also be noted that the proposed 389 slices represent a very sparse sampling of the azimuthal sub-³⁹⁰ space and as such, can omit some reflectance features, resulting ³⁹¹ in a slightly different color/brightness appearance of the recon-³⁹² structed data. To avoid this problem, the azimuthal subspace ³⁹³ can be sampled by additional slices at the cost of slightly longer ³⁹⁴ acquisition times. Second, the interpolation step of the algo-395 rithm expects monotonicity of reflectance values across differ-396 ent illumination and view elevations. However, this condition 397 is rarely invalid and no such behavior was experienced with any 398 of the tested materials. The method's accuracy can be further ³⁹⁹ improved in this respect by taking more slices at different eleva-400 tions. Finally, highlights of extremely specular samples are not 401 always represented accurately enough (see Fig. 19) mainly due 402 to an insufficient angular sampling of azimuthal angles (step $_{403}$ 15°) in original datasets used in the experiments. Note that the 404 sampling density along specular highlights in diagonal slices 405 can be arbitrarily increased to provide a better match of specu-406 lar highlights of a high-dynamic-range within the model with-407 out increasing the measurement time.

Note that the proposed method does not fit any analytical 409 model to the measured data and as such it is sensitive to noise 410 in the measurement process. However, since the measurement 411 procedure is fast and simple, this noise can be effectively sup-412 pressed by measuring the slices several times and computing 413 the measurements' median values.

414 6. Sparse ABRDF Data Measurement

This section describes a practical experiment of capturing 416 sparse ABRDF samples using a consumer camera and a LED 417 point-light source and is followed by a complete ABRDF re-418 construction from such measurements.

Mutual movement of arms with camera and light with re-420 spect to the sample being measured is controlled manually as is shown in Fig. 14. The axial slice s_A data (left) are measured



Figure 14: The proposed ABRDF measurement setup at fixed elevation angles θ_i/θ_v . 421

422 using rotation of the fixed light and sensor around the sample,

²http://vision.ucsd.edu/kriegman-grp/research/vst/



Figure 12: Comparison of the material's BRDF (the first row), and interpolation of missing values by means of method A (the second row), method B (the third row), and its modification method BD (the fourth row) respectively. Below are 10× difference images and global difference values in CIE $\Delta E / RMSE / PSNR[dB]$.



9.4/13.4/25.6 9.8/13.4/25.6 13.6/22.2/21.3 11.4/18.0/23.0 24.1/20.6/21.9 11.0/22.7/21.0 8.8/13.9/25.3 17.1/27.3/19.4 9.8/13.7/25.4 Figure 13: The performance of interpolation from 169 sparse uniform samples (13 samples per hemisphere). Below are the difference values in: CIE $\Delta E / RMSE / PSNR[dB].$

424 ally opposite movements of the light and sensor in respect to 432 tion set shown in Fig. 15-b. During its movement, the camera 425 the sample. Both the camera and light travel full circle around 433 records the material sample appearance as a video sequence at ⁴²⁶ the sample and return to the initial position.

427 $_{428}$ Lumix DMC-FT3 and light using high-power LED Cree XLamp $_{436}$ b. Both s_A and s_D slices are recorded for two different ele-429 XM-L with 20° frosted optics (Fig. 15-a). To achieve the re-430 quired synchronous movement of light and camera, we con-

 $_{423}$ while the diagonal slice s_D data (right) are obtained by mutu- $_{431}$ structed a frame with two arms using a Merkur toy³ construc-434 a resolution of 1280×720 pixels, and the elevation angles of Our acquisition setup consisted of the Panasonic camera 435 the camera and light are kept constant using the setup Fig. 15-

³http://www.merkur.cz/



Figure 15: Data acquisition equipment (a) with its fixating frame (b), and measured sample with registration borders for calibration (c).

437 vations of the camera (C1, C2) and light (L1, L2); therefore, ⁴³⁸ eight slices are measured approximately at elevations $\theta_i/\theta_v =$ 439 [30°/30°, 30°/75°, 75°/30°, 75°/75°] as shown in Fig. 3-b. Record 495 samples (14²), while the remaining samples are interpolated us-440 ing of the slices took less than 10 minutes. From each of the 441 eight video sequences, 24 frames were extracted corresponding ⁴⁴² to sampling of azimuthal angles φ_i/φ_v every 15°. This resulted 443 in a total of 192 samples being obtained. The number differs 444 from 168 samples used in the reconstructions in Section 4, be-445 cause this time all elevations were covered by the same num-446 ber of samples. The effective number of samples is always 447 slightly lower than 192, as some of the frames are removed 448 due to occlusion of the material by the arm with light. Note 449 that the method's principle allows adaptive density of the sam-450 ples (frames) along the slices to also record extremely narrow 451 specular highlights.

Three anisotropic fabric materials (30×30 mm) were used 452 453 as test samples, as shown in Fig. 16. A white border was at-454 tached around the sample to help detect camera orientation in 455 respect to the sample coordinate space and for sample registra-⁴⁵⁶ tion Fig. 15-c). Because of this, we first calibrated the camera 457 [31]. Unfortunately, the used low-end camera adapts its expo-⁴⁵⁸ sure depending on the amount of light coming from the scene. 459 On the other hand, this feature enables us to capture as much 460 information as possible, even using a limited dynamic range of ⁴⁶¹ the camera's sensor (8bits/color). Since the information about 462 exposure throughout the video sequence could not be retrieved 463 from an EXIF header as is possible for still photos, we used the 464 reference BRDF data of dark material surrounding the sample 465 to compensate for exposure of each image. That is, we com-⁴⁶⁶ pensated color values of the sample using the black part of the ⁴⁶⁷ sample holder (near the white borders as shown in Fig. 15-c) 468 and its reference measurements.

Subsequent processing was then performed for each image. 469 ⁴⁷⁰ Camera viewing angles θ_v/φ_v were obtained from camera ex-471 trinsic parameters, given the known camera calibration and cor-472 ner points of the white borders. Coordinates of these points ⁴⁷³ were obtained from the image registration based on the camera 474 calibration. When the viewing angles were known, the illumi-⁴⁷⁵ nation azimuth angle was computed as: $\varphi_i = \varphi_v - \alpha$ for the axial ⁴⁷⁶ slice s_A , and $\varphi_i = 2\pi - \varphi_v$ for the diagonal slice s_D . The elevation 477 angles θ_i were estimated from the slant of the light during mea-⁴⁷⁸ surement. Finally, the slice's ABRDF value from each image 479 was obtained as the average of RGB values near the sample's 480 center, and colorimetrically calibrated. The non-optimized data ⁴⁸¹ processing described above took approximately 10 minutes to 482 perform over all selected images. The reference ABRDF mea-483 surements of the black target and materials are obtained from

⁴⁸⁴ the UTIA BTF database⁴ and have the same angular resolution 485 as BTF Database Bonn [29].

When all of the selected images were processed in this way 487 and data for all eight slices were obtained, the ABRDF space re-488 construction described in Sections 3.2 and 3.3 was performed. 489 Figure 17 compares reference ABRDF measurements of the ⁴⁹⁰ material (a) with their reconstruction from the 192 sparse ref-491 erence samples using Method B (b), and with a reconstruction ⁴⁹² using 192 sparse measurements obtained by the proposed setup 493 and interpolation Method B (c). The last column (d) of Fig-⁴⁹⁴ ure 17 compares our method with a uniform sampling using 196 496 ing Method A (compare with column (b)). Note that, while the 497 visual performance of the uniform sampling might look simi-⁴⁹⁸ lar, the complexity of its measurement is considerably higher in ⁴⁹⁹ comparison with the proposed measurement approach.



Figure 16: Three anisotropic fabric samples whose ABRDFs were measured using the proposed setup.

500 Finally we took photographs of the fabric02 and fabric03 ⁵⁰¹ materials attached on a cylinder (a) and compared them with ⁵⁰² renderings on a cylinder using their reference BRDFs (b) and ⁵⁰³ BRDFs captured by the proposed setup (c). The results for dif-⁵⁰⁴ ferent illumination conditions are shown in Fig. 18 and con-505 firm that even the proposed approximate measurement setup 506 can record BRDFs with reasonable accuracy, in comparison to ⁵⁰⁷ the reference measurements.

508 The reconstruction results from our preliminary measure-⁵⁰⁹ ments (Figure 17-c) are encouraging and we believe that they 510 convey the idea of ABRDF capturing speed and simplicity with-⁵¹¹ out the need for dedicated and thus costly devices.

512 A notable advantage of our setup and the proposed sampling 513 pattern is its ability to quickly measure any flat samples without ⁵¹⁴ needing to extract them from their environment, and thus it can 515 be used for fast and inexpensive measurements of such samples 516 as human skin and precious cultural heritage objects.

517 7. Experimental BTF Reconstruction

As the acquisition and reconstruction of spatially-varying 518 519 datasets is a straightforward extension of the proposed sparse 520 sampling and reconstruction method, we tested the method's $_{521}$ performance on ten BTF samples of angular resolution 81 \times $_{522}$ 81 = 6561 images as described in Section 4. Only 168 im-523 ages (corresponding to the eight data slices) were selected from 524 the BTF samples and used for pixel-wise reconstruction of the 525 remaining images using the proposed method.

⁴http://btf.utia.cas.cz



Figure 17: ABRDF reference measurement (a), compared to reconstruction from 168 sparse reference measurements only using Method B (b), and reconstruction from the proposed measurement procedure using 192 samples and interpolation Method B (c), uniform interpolation using 196 samples (d). Below are the difference values in: CIE $\Delta E / RMSE / PSNR[dB]$.

526 7.1. Results

⁵²⁷ Renderings of the original data with results of the proposed ⁵²⁸ reconstruction methods for point-light and environment illu-⁵²⁹ mination is shown side-by-side in Figures 19 and 20, respec-⁵³⁰ tively. All differences are objectively compared using CIE ΔE , ⁵³¹ PSNR[dB], SSIM [30], and VDP2 [18] metrics. From the re-⁵³² sults it is apparent that for samples with lower height variations, ⁵³³ there is a close match to the original data. The apparent devia-⁵³⁴ tions from the original data for materials having higher surface ⁵³⁵ height variations are caused mainly by the incorrect geometry ⁵³⁶ preservation of structural elements.

537 7.2. Limitations

Although there are not any restrictions imposed on view and big illumination dependent datasets, the results have shown that big the BTF reconstruction is incorrect for those materials which



Figure 18: Photographs of the *fabric01* and *fabric02* samples on a cylinder illuminated from top, bottom, left, and right (a) compared with renderings using reference ABRDF (b), and sparsely measured and reconstructed ABRDF (c).

541 have a wide range of surface height variation, e.g., corduroy 542 and Lego samples shown in Fig. 21. This is caused partly by 543 very sparse sampling of the azimuthal space, as well as by in-544 terpolation of the data at missing elevations. While the former 545 produces geometrical deformation of the structure's features, 546 the latter causes their blur as well as improper highlights ex-547 trapolation for low elevation angles. Even though the recon-548 struction from sparse samples for such materials is not accurate 549 in terms of correct shading of structure elements, the method 550 correctly captures the look-and-feel of the material's spatially-⁵⁵¹ varying appearance for nearly-flat samples, e.g., for *fabric dark*, 552 fabric light, and leather light samples. However, in compar-553 ison with the SVBRDF measurement and representation ap-554 proaches, the proposed method is not limited to restrictions im-555 posed by BRDF itself. Therefore, it may be found useful for 556 guick, low-cost, and fairly accurate acquisition and BTF recon-557 struction of many materials having a limited height variation, 558 e.g., fabric and leather.

The time of BTF data reconstruction depends only on its static spatial resolution, since individual pixels are regarded as independent ABRDFs. Due to huge sizes of datasets, only repetitive BTF tiles were used. While reconstruction of a single pixel took sea \approx 1 second, the non-optimized reconstruction of a BTF tile of size 128² took 4.5 hours using the 3 cores of the Intel Xeon 565 2.7 GHz. Therefore, using optimized multi-core CPU's imple-566 mentation processing times of less than one hour can be easily 567 achieved.

Note that the proposed sparse acquisition and reconstruction method is complementary to BTF compression methods. In always processing an entire BTF dataset, any of these methtods can be applied to compress the reconstructed data. By its sparse measurement, our method can achieve a compression ratio 1:39; however, in terms of reconstruction quality and compression ratio, it cannot compete with BTF compression approaches, as seen in the local PCA method [21] (using 5 clus-



 $\frac{8.1}{28.1} + \frac{28.1}{0.88} + \frac{83.7}{0.4} = \frac{7.4}{26.9} + \frac{26.9}{0.84} + \frac{83.5}{0.42} = \frac{11.6}{22.0} + \frac{20.0}{0.85} + \frac{81.9}{0.4} = \frac{10.4}{23.4} + \frac{20.0}{0.69} + \frac{20.0}{79.5}$ Figure 19: A comparison of BTF rendering from the full dataset of 6561 images (the first row), with its reconstruction from only 168 images (the second row) in single point-light illumination. Below are the CIE $\Delta E / PSNR[dB] / SSIM / VDP2$ difference values.

⁵⁷⁶ ters, 5 components) in Tab. 1 (compare with our reconstruction ⁵⁷⁷ errors in Fig. 19). The variable compression ratio of the local ⁵⁷⁸ PCA method is due to the variable size of the BTF tile used.

579 8. Conclusions

A novel method of sparse measurement and reconstruction of view and illumination dependent datasets has been proposed. The proposed sparse sampling of illumination and viewing directions allows for intuitive continuous measurement by a consumer camera and LED light. The reconstruction from such sparse data does not impose any restrictions on input data and allows reliable approximation of anisotropic non-reciprocal view and illumination dependent datasets. Additionally, this method ing and outgoing directions. The method's performance was tested on isotropic BRDFs and anisotropic apparent BRDFs

Table 1: A reconstruction error and compression ratio of LPCA compression method.

material	$\Delta E / PSNR[dB] / SSIM / VDP2$				C.R.	(tile)
alu	3.5	34.6	0.99	93.8	18.2	(21×26)
corduroy	2.7	37.5	0.97	93.6	55.0	(36×46)
fabric d.	4.6	32.2	0.94	86.2	16.1	(21×23)
fabric l.	8.6	26.3	0.95	89.4	10.4	(19×23)
leather d.	4.3	33.3	0.97	91.0	263.0	(93×86)
leather l.	10.8	24.3	0.95	87.7	193.0	(74×79)
l. wood	11.3	23.5	0.92	81.0	627.8	(137×142)
k. wool	4.3	32.1	0.96	93.0	20.8	(25×25)



5.1 / 33.2 / 0.96 / 90.0 3.4 / 34.6 / 0.96 / 91.1 8.9 / 26.6 / 0.91 / 84.3 4.6 / 30.7 / 0.88 / 86.5 Figure 20: A comparison of BTF rendering from the full dataset of 6561 images (the first row), with its reconstruction from only 168 images (the second row) in grace environment illumination [5]. Below are the CIE $\Delta E / PSNR[dB] / SSIM / VDP2$ difference values.

⁵⁹² periments have shown that retrieval of sparse samples and the ⁵⁹³ consequent reconstruction of the complete dataset take less than ⁵⁹⁴ half an hour. Experimental sparse reconstruction of BTF datasets ⁶¹⁰ Acknowledgments 595 has shown that the method can be a reasonably accurate alternative to lengthy measurement, especially for samples having a ⁵⁹⁷ smaller height variation. The ease of data acquisition and visual ⁵⁹⁸ quality of the reconstruction using this method makes it supe-⁵⁹⁹ rior to alternative approaches such as bump/displacement map-600 ping or parametric BRDF modeling. Because of the simplicity 601 of data acquisition and reconstruction, this approximate method 602 can be utilized in less accuracy-demanding applications. Since 603 digital reproduction of a material's appearance look-and-feel 604 can be created inexpensively, it could be particularly useful in 618 [2] Nicolas Bonneel, Michiel van de Panne, Sylvain Paris, and Wolfgang Hei-605 the fields of computer gaming, film and digital presentations of 606 e-commerce.

In summation, we believe this research will contribute to 60 608 future development of simple, inexpensive, and portable acqui-

⁵⁹¹ with encouraging results. Our pilot ABRDF measurement ex- ⁶⁰⁹ sition setups of illumination and view dependent data.

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Figure 21: A failure case of the proposed BTF reconstruction for rough materials: *corduroy* and *Lego*. Below are the CIE ΔE / PSNR[dB] / SSIM / VDP2 difference values.

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