Mathematical continuum mechanics

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This course will be mainly devoted to a mathematical treatment of elasticity and viscoelasticity. We will start with a basic formulation of the problem, with notions of deformation and displacement, and with the axiom of the existence of the stress vector in elastic solids. Further, we are going to discuss Lagrangean description, deformation gradients, measures of deformation and transformation rules for line, surface and volume elements. We obtain equilibrium equations in the reference and deformed configurations of the body and discuss various definitions of stress (Cauchy, Piola-Kirchhoff, Kirchhoff). We will show weak convergence of minors of deformation gradients and prove the existence of equilibrium states for polyconvex materials with some additional restrictions (Signorini contact problems, for instance). We then formally derive linearized elasticity theory from the large-strain setting.

The second part of the course will focus on evolutionary problems as, e.g., elastoplasticity and models of viscoelastic materials (Maxwell, Kelvin-Voigt). If time permits we mention coupling with thermal processes, too.

We will assume some basic knowledge of Lebesgue and Sobolev spaces as well as of elementary linear functional analysis (Hahn-Banach theorem, weak convergence, Riesz theorem, Lax-Milgram lemma). No physical background is needed but may be advantageous.

Some references

[1] Antman, S.S.: Nonlinear Problems of Elasticity, Springer, New York, 1995.
[2] Ciarlet, P.G.: Mathematical Elasticity. North-Holland, Amsterdam, 1988.
[3] Haupt, P.: Continuum mechanics and theory of materials. Springer, 2000.
[4] Horgan, C.O.: Korn's inequalities and their applications in continuum mechanics. SIAM Rev. **37** (1995), 491–511.
[5] Šilhavý, M.: The Mechanics and Thermodynamics of Continuous Media. Springer, 1997.