Special topics in the calculus of variations

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Our seminar is going to be focused on questions appearing when "mathematics meets mechanics", i.e., when we want to apply tools of mathematical analysis and calculus of variations to problems of continuum mechanics of solids. We will see that natural physical requirements can become rather complicated for mathematical modeling.

We will start with the role of convexity in the variational calculus and show importance of Mazur's lemma. Weak convergence in Lebesgue spaces will become an important tool and we will obtain much more information from weakly converging sequences than a mere limit. The main instrument for us will be parametrized (Young) measures. Then we will introduce more general notions of convexity used in vectorial multi-dimensional problems. We will show how standard proofs of weak lower semicontinuity do not allow for physically natural constraints as orientation preservation of elastic deformations, for instance. We will also indicate some new techniques how to overcome this obstacle in some particular cases.

If time permits, we introduce a notion of Γ-convergence, prove some of its properties, and we also emphasize its applications to mechanical problems as dimension reduction or treatment of singular perturbations.

We will assume some basic knowledge of Lebesgue and Sobolev spaces as well as of elementary linear functional analysis (Hahn-Banach theorem, weak convergence). No physical background is needed but may be advantageous.

Some references

[1] Braides, A.: Γ-Convergence for Beginners. Oxford University Press, Oxford, 2002.
[2] Dacorogna, B.: Direct Methods in the Calculus of Variations. 2nd ed., Springer, New York, 2008.
[3] Evans, L.C.: Weak Convergence Methods for Nonlinear Partial Differential Equations, CBMS 74, American Mathematical Society, 1990. Third printing, 2002.