Program

Tuesday, March 25
14:00 Opening
14:10 T. Iwaniec: Limits of Sobolev homeomorphisms and existence of 2D-traction free minimal deformations
15:00 S. Hencl: Optimal assumptions for discreteness
15:50 coffee break
16:10 S. Conti: On the theory of relaxation with a determinant constraint

Wednesday, March 26
10:00 B. Dacorogna: A review on the Jacobian equation
10:50 F. Rindler: The positive Jacobian constraint in Sobolev spaces with integrability below the dimension and orientation-preserving Young measures
11:40 coffee break
12:00 B. Benešová: Quasiconvexity conditions when minimizing over homeomorphisms in the plane

14:30 H. Le Dret: Graphene sheets with three-point interactions
15:20 O. Anza-Hafsa: Homogenization of unbounded integrals with quasiconvex growth
16:10 coffee break
16:30 S. Neukamm: Homogenization of nonlinear bending plates

20:00 Workshop dinner: Le Martel, 3 Rue Martel, Paris 10

Thursday, March 27
10:00 J. Kristensen: Rankone convexity and integral estimates
10:50 S. Krömer: Signed integral functionals with linear growth: Weak* lower semicontinuity in BV without prescribed boundary values
11:40 coffee break
12:00 E. Spadaro: Lower semicontinuous functionals defined on spaces of multiple-valued maps

14:15 J.-P. Mandallena: On relaxation, homogenization and 3d-2d passage with determinant type constraints
15:05 M. Barchiesi: A nonlinear model for nematic elastomers
15:55 D. Henao: Continuity of the Jacobian and surface energies in the analysis of cavitation
16:45 Closing

Abstracts

Limits of Sobolev homeomorphisms and existence of 2D-traction free minimal deformations

Tadeusz Iwaniec
Syracuse University

Let $\mathbb{X}, \mathbb{Y} \subset \mathbb{R}^2$ be bounded Lipschitz domains of the same topological type. We prove that the weak and strong limits of homeomorphisms $h : \mathbb{X} \overset{\text{onto}}{\to} \mathbb{Y}$ in the Sobolev space $\mathscr{W}^{1,p}(\mathbb{X}, \mathbb{R}^2)$, $p \geq 2$, are the same. Moreover, these limits are none other than the *Monotone Sobolev Mappings*. Uniform approximation of monotone mappings with homeomorphisms between compact metric spaces is of great interest in topology. In the Sobolev setting such approximation problems are at the very heart of Geometric Function Theory and Nonlinear Hyperelasticity. As an application, we establish the existence of 2D-traction free minimal deformations (slipping along the boundary is allowed) for fairly general energy-integrals. When passing to the weak (strong) limit of the minimizing sequence of homeomorphisms the collapse of injectivity (interpenetration of matter, also interpreted as formation of cracks) may occur. This is joint work with Jani Onninen.

Optimal assumptions for discreteness

Stanislav Hencl
Charles University

In the pioneering works J. Ball studied a class of mappings that could be used to model nonlinear elasticity and he found weak conditions for regularity and invertibility properties. One of the main properties in the models of nonlinear elasticity is that there is no interpenetration of matter. This in the physically relevant models corresponds to the fact that two parts of the body cannot be mapped to the same place. From the mathematical point of view this means that the map is one-to-one and thus invertible.

Let $\Omega \subset \mathbb{R}^n$ and $f : \Omega \to \mathbb{R}^n$ be a mapping. In this talk we discuss the optimal assumptions that guarantee that the mapping is open and discrete which is the main ingredient for showing that it is invertible and in fact a homeomorphism. The optimal condition in the plane for mappings of finite distortion was found by Iwaniec and Šverák in 1993 and they conjectured the optimal assumption in higher dimension. Many people have contributed to the study of this problem and recently we have found out that there is a counterexample to dicreteness in dimension $n \geq 3$. This is a joint result with Kai Rajala.

On the theory of relaxation with a determinant constraint

Sergio Conti
Universität Bonn

We consider vectorial variational problems in nonlinear elasticity of the form $E[u] = \int W(Du)$, with the nonlinear side condition $\det Du = 1$ or $\det Du > 0$ almost everywhere. Under suitable growth and continuity assumptions we show that, if the quasiconvex envelope W^{qc} of W is polyconvex, then the functional $\int W^{\mathrm{qc}}(Du)$ coincides with the relaxation of E. This is joint work with Georg Dolzmann.

A review on the Jacobian equation

Bernard Dacorogna
Ecole Polytechnique Fédérale de Lausanne

I will discuss the equation $\det Du = f$.
I will consider
1) First the case where $f > 0$;
2) Then the case where f can change sign;
3) Finally the case where we have several equations of the above type.

The positive Jacobian constraint in Sobolev spaces with integrability below the dimension and Young measures

Filip Rindler
University of Cambridge

The main difficulty in manipulating a map satisfying the constraint that the (pointwise) Jacobian determinant be positive or even a-priori prescribed is due to the strongly non-linear and non-convex nature of this condition. In this talk, which is based on joint work with K. Koumatos (Oxford) and E. Wiedemann (UBC/PIMS), I will present various recent results on how such function can be manipulated under preservation of this constraint, at least in in subcritical Sobolev spaces, where the integrability exponent is less than the dimension. In particular, I will give a characterization theorem for Young measures under this side constraint. This is in the spirit of the celebrated Kinderlehrer–Pedregal Theorem and based on convex integration and "geometry" in matrix space. Finally, applications to the minimization of integral functionals, the theory of semiconvex hulls, and approximation in Sobolev spaces are given.

Quasiconvexity conditions when minimizing over homeomorphisms in the plane

BARBORA BENEŠOVÁ
RWTH Aachen

In this talk we characterize necessary and sufficient conditions on the stored energy density in order to assure weak-$\star$ lower semicontinuity on the set of bi-Lipschitz functions in the plane. This problem is motivated by variational problems in nonlinear elasticity where the orientation preservation and injectivity of the admissible deformations are key requirements. Generally speaking, the main difficulty in finding such conditions is that the set of bi-Lipschitz functions is non-convex. Thus, standard cut-off techniques that modify the generating sequence to have the same boundary conditions as the limit generally fail; however, the standard proofs of in calculus of variations rely on such methods. We obtain this cut-off by following a strategy inspired by Daneri & Pratelli, i.e. we modify the generating sequence (on a set of gradually vanishing measure near the boundary) first on a one dimensional grid and then rely on bi-Lipschitz extension theorems. We also present method of modifying the sequence on the grid that could be extended to more general classes of mappings.

Graphene sheets with three-point interactions

HERVÉ LE DRET
Université Pierre et Marie Curie

We present a discrete to continuum approach for graphene sheets with three-point interactions due to torques between adjacent chemical bonds. This is ongoing work with Annie Raoult.

Homogenization of unbounded integrals with quasiconvex growth

OMAR ANZA-HAFSA
Université de Nîmes

Nonconvex homogenization by Γ-convergence was mainly studied in the framework of p-polynomial growth on the integrand, but these conditions are not compatible with basic conditions of hyperelasticity. We are interested to go beyond the polynomial growth by studying homogenization with quasiconvex growth on the integrand.

Homogenization of nonlinear bending plates

Stefan Neukamm
Weierstraß-Institut for Applied Analysis and Stochastics, Berlin

Starting from a 3d nonlinearly elastic composite (with periodic microstructure), we derive effective plate models in the bending regime by simultaneously passing to the zero-thickness- and homogenization limit. The limiting model features a dependence on the relative scaling between the thickness and the size of the microstructure. The derivation is based on a detailed description of the oscillatory structure of the nonlinear strain tensor, which we explore with help of two-scale convergence. If time permits, we address in a second part the homogenization problem of a nonlinear plate model with reduced dimension. Again, the result is based on a detailed description of the oscillatory structure of the second fundamental form associated with pure bending deformations.

Rank–one convexity and integral estimates

Jan Kristensen
University of Oxford

It is known that many interesting questions about sharp integral estimates involving derivatives of mappings can be reformulated as questions about quasiconvexity properties of associated integrands. In this talk we discuss the strong convexity properties of one–homogeneous rank–one convex integrands and some of their consequences for integral estimates.

The talk is based on joint work with Bernd Kirchheim (Leipzig).

Signed integral functionals with linear growth: Weak-$\star$ lower semicontinuity in BV without prescribed boundary values

Stefan Krömer
Universität zu Köln

Motivated by the study of a suitable generalization of gradient Young measures suitable for functionals in BV, J. Kristensen and F. Rindler obtained a characterization of weak-$\star$ lower semicontinuity for functionals in BV with possible linear growth of the negative part. So far, this only works for sequences with fixed boundary values, or for functionals incorporating boundary values via a jump term at the boundary of the domain, effectively limiting the applications to problems subject to Dirichlet boundary conditions. I will present recent results without this restriction, which lead to additional requirements on the energy density related to possible concentration effects of sequences near the boundary.

This is joint work with B. Benešová (Aachen) and M. Kružík (Prague).

Lower semicontinuous functionals defined on spaces of multiple-valued maps

Emanuele Spadaro
Max-Planck-Insitute for Mathematics in the Sciences, Leipzig

In this talk I will discuss a class of energies defined on spaces of functions taking more than one value. Although the interest in this kind of energies stemmed from the geometric analysis, they can be also used to suggest novel multi-scale and multi-field descriptions of material microstructures.

After a brief introduction to the motivations and the connections to material science, I will introduce precise definitions and present some results on the semicontinuity of such energies.

On relaxation, homogenization and 3d-2d passage with determinant type constraints

Jean-Philippe Mandallena
Université de Nîmes

Some results on relaxation, homogenization and 3d-2d passage in the framework of the multidimensional calculus of variations and related to hyperelasticity are presented.

A nonlinear model for nematic elastomers

Marco Barchiesi
Università degli Studi di Napoli

I will discuss the well-posedness of a new nonlinear model for nematic elastomers. The main novelty is that the Frank energy penalizes spatial variations of the nematic director in the deformed, rather than in the reference configuration, as it is natural in the case of large deformations.

Continuity of the Jacobian and surface energies in the analysis of cavitation

Duvan Henao
Facultad de Matemáticas
Pontificia Universidad Católica de Chile

Jointly with Carlos Mora-Corral (Univ. Autónoma de Madrid) and Xianmin Xu (Chinese Academy of Sciences), we have studied ([1]–[7]) the problem of cavitation in nonlinear elasticity, under conditions allowing for the initiation and propagation of classical codimension-one fractures, that is, in SBV. In order to recover the weak continuity of the Jacobian we add a surface energy term that penalizes the creation and the stretching of new surface:

$$\mathcal{E}(\mathbf{u}) = \sup_{\substack{\mathbf{f}\in C_c^\infty(\bar\Omega\times\mathbb{R}^n;\mathbb{R}^n)\\ \|\mathbf{f}\|_\infty\le 1}} \int_{\mathbf{u}(\Omega)} \operatorname{div}_{\mathbf{y}}[\mathbf{f}(\mathbf{u}^{-1}(\mathbf{y}),\mathbf{y})]\,\mathrm{d}\mathbf{y},$$

which is essentially equivalent to the perimeter of the deformed configuration

$$\operatorname{Per}\mathbf{u}(\Omega) = \sup_{\substack{\mathbf{g}\in C_c^\infty(\mathbb{R}^n)\\ \|\mathbf{g}\|_\infty\le 1}} \int_{\mathbf{u}(\Omega)} \operatorname{div}\mathbf{g}(\mathbf{y})\,\mathrm{d}\mathbf{y}$$

proposed by Müller & Spector (1995) (though, for technical reasons and due to pathological examples, in SBV it is necessary to consider test functions of two variables).

In this talk I will highlight the main ideas in the proof of our results, in particular the weak continuity of the Jacobian under an uniform bound on the surface energy and the convergence of an Ambrosio-Tortorelli type of regularization,

$$\min_{\substack{\mathbf{u}\in W^{1,p},\operatorname{Det}D\mathbf{u}=\det D\mathbf{u}\\ v\in W^{1,q}(\Omega),w\in W^{1,q}(Q)}} \int_\Omega (v(\mathbf{x})^2+\eta)W(D\mathbf{u}(\mathbf{x}))\,\mathrm{d}\mathbf{x} + \lambda_1\int_\Omega\left[\varepsilon^{q-1}\frac{|Dv(\mathbf{x})|^q}{q} + \frac{(1-v(\mathbf{x}))^{q'}}{q'\varepsilon}\right]\mathrm{d}\mathbf{x}$$

$$+\,6\lambda_2\int_Q\left[\varepsilon^{q-1}\frac{|Dw(\mathbf{y})|^q}{q} + \frac{w(\mathbf{y})^{q'}(1-w(\mathbf{y}))^{q'}}{q'\varepsilon}\right]\mathrm{d}\mathbf{y},$$

that makes it possible to numerically study the initiation of fracture by void coalescence.

References

[1] D. Henao, Cavitation, invertibility, and convergence of regularized minimizers in nonlinear elasticity. *J. Elast.* **94** (2009) 55–68.

[2] D. Henao, C. Mora-Corral, Invertibility and weak continuity of the determinant for the modelling of cavitation and fracture in nonlinear elasticity. *Arch. Ration. Mech. Anal.* **197** (2010) 619–655.

[3] D. Henao, C. Mora-Corral, Fracture surfaces and the regularity of inverses for *BV* deformations. *Arch. Ration. Mech. Anal.* **201** (2011) 575–629.

[4] D. Henao, C. Mora-Corral, Lusin's condition and the distributional determinant for deformations with finite energy. *Adv. Calc. Var.* **5** (2012) 355–409.

[5] D. Henao, X. Xu, An efficient numerical method for cavitation in nonlinear elasticity. *Math. Models Methods Appl. Sci.* (2011) 1733–1760.

[6] D. Henao, C. Mora-Corral, X. Xu, Γ-convergence approximation of fracture and cavitation in nonlinear elasticity (in preparation).

[7] D. Henao, C. Mora-Corral, X. Xu, A numerical study of void coalescence and fracture in nonlinear elasticity (in preparation).