Tools for Communication of Bayesian Agents

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12th October 2005
Outline

1. Introduction to Multi-agent Systems
   - Example: temperature control
   - Issues of multi-agent systems

2. Bayesian Decision Making
   - Adaptive Bayesian Decision-Maker
   - Towards Bayesian Agents
   - Key technologies

3. Merging of Ideal Pdfs
   - Merging of Ideal Pdfs - Problem Formulation
   - Solution

4. Conclusions
Example: temperature control

Fictitious room:

Task:
control the room temperature

reliably: failures,
adaptively: changes in the environment
Example: temperature control

Fictitious room:

Centralized control:
- optimization
- many possible scenarios
- poor scalability
- error sensitive
- poor reconfiguration
Example: temperature control

Fictitious room:

Agent control:

**scalable**: agents can be added

**simple**: few rules

**cheap**: agents in devices

**expensive**: communication

Industrial standard: Rockwell automation
Centralized vs. Decentralized Control

Centralized approach:
- Has a *consistent* theory of decision-making under uncertainty (Bayesian theory),
- Faces the “curse of dimensionality”, solution for complex problems is prohibitive,
- Re-design is not flexible enough and requires a lot of manpower,

Distributed Approach (Multi-agent):
- Complex problem is decomposed into local areas which are governed by autonomous agents,
- The agents communicate to each other to achieve overall coordination,
- It is difficult to assess the overall behaviour of the MAS, (game theory),
Centralized vs. Decentralized Control

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- Has a consistent theory of decision-making under uncertainty (Bayesian theory),
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- Re-design is not flexible enough and requires a lot of manpower,

Proposal: take best of those worlds.

Distributed Approach (Multi-agent):

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Multiple Participant Decision Making = Bayesian Agents
Solid \textit{consistent} theory of making decisions under \textit{uncertainty}.

Decision-maker is using probability calculus:

\textbf{Model}: \( f(d(t), \Theta(t)) \), relation of data and parameters.

\textbf{Aim}: \( f(d(t), \Theta(t)) \), ideal distribution,

\textbf{Decision}: \( f(u_t|d(t)) \rightarrow u_t \), optimal decisions.
How to make a Bayesian Agent?

Making Bayesian decision-maker aware of each other

**Communication** exchange of information $\Rightarrow$ better learning,

**Cooperation** exchange of aims (pdfs) $\Rightarrow$ avoiding conflicts.
How to make a Bayesian Agent?

Making Bayesian decision-maker aware of each other

Communication exchange of information ⇒ better learning,

Cooperation exchange of aims (pdfs) ⇒ avoiding conflicts.

The task:
Formalization in terms of probability calculus and algorithmic solution.
Fully probabilistic design:

the aim of decision making is formalized in the form of ideal distribution,

\[ f (d (t), \Theta (t)) . \]

- the loss function of divergence between the ideal and the true pdf.
  Advantage: **no need to exchange loss functions!**
- Optimal strategy is known: \( f (u_t|d(t)) = \int \int \int \ldots \)
- Allows for multi-criteria decision-making
- Solvable for Markov chains and Gaussian pdf, otherwise approximations.
Key technologies: Merging

Merging of probability distributions: (information fusion)
Key technologies: Merging

Merging of probability distributions: (information fusion)

![Graph](image-url)
Key technologies: Merging

Merging of probability distributions: (information fusion)

- various types of pdfs (Gauss, discrete, etc.),
- on different variables, of different type (marginalized, conditioned)
Projection:

- finding ‘nicer’ distribution, loosing as little information as possible
Example: Negotiation of temperature

Classical agents:

A1 (cooling): goal 15 °C
A2 (heating): goal 20 °C

scenarios:

1. non-cooperating agents: 18 °C, both are working on full steam,
2. fully cooperating agents: 18 °C, lower energy load.

Negotiation: mostly ad hoc methods
Example: Negotiation of temperature

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Bayesian agents:

- **A1 (cooling):** $\mathcal{I}(T) = \mathcal{N}(15, 2)$
- **A2 (heating):** $\mathcal{I}(T) = \mathcal{N}(20, 6)$
Example: Negotiation of temperature

Bayesian agents:

A1 (cooling): \( \mathcal{N}(15, 2) \)
A2 (heating): \( \mathcal{N}(20, 6) \)

scenarios:

1. non-cooperating agents: same
2. fully cooperating agents:
   \( \mathcal{N}(17, 7) \), result of optimization.

Negotiation: faster convergence, lower communication load.
Problem Formulation

- vector random quantity
  \[ x = (q_1, \ldots, q_N) \]
- \( n \) agents, ideal pdfs \( f_p(x_p) \)
- \( x_p \) – random vectors, entries from \( \{q_1, \ldots, q_N\} \)
- weights \( \alpha_p > 0, \sum_p \alpha_p = 1 \)
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**Common ideal pdf \( f(x) \)?**
- How to define \( f(x) \)?
- How to find it?
- Practical issues
Common ideal pdf

\[ f(x) \in \arg \min_{\tilde{f}} \sum_p \alpha_p D(f_p(x_p) \| \tilde{f}(x_p)) \]

\( D(\cdot \| \cdot) \) - Kullback-Leibler divergence
Solution

Common ideal pdf

\[ f(x) \in \arg \min_{\tilde{f}} \sum_{p} \alpha_p D(f_p(x_p)||\tilde{f}(x_p)) \]

\[ f(x) = \sum_{p} \alpha_p \frac{f(x)}{f(x_p)} f_p(x_p) \]

\( D(\cdot||\cdot) \) - Kullback-Leibler divergence
Approximation of Common Ideal Pdf

\[ D(h) = \sum_p \alpha_p D(f_p(x_p) \mid \mid h(x_p)) \]

\[ A(h) = \sum_p \alpha_p \frac{f(x)}{f(x_p)} f_p(x_p) \]
Approximation of Common Ideal Pdf

\[
\mathcal{D}(h) = \sum_p \alpha_p D(f_p(x_p) \| h(x_p))
\]

\[
A(h) = \sum_p \alpha_p \frac{f(x)}{f(x_p)} f_p(x_p)
\]

\[
\mathcal{D}(h) \geq \mathcal{D}(Ah) \quad \forall h
\]

\[
\mathcal{D}(h) = \mathcal{D}(Ah) \quad \text{iff } h \text{ is optimal}
\]

\[
\mathcal{D}(A^k h) \to \mathcal{D}(f)
\]
Practical Issues

- discrete quantities
  - directly usable
  - marginalization computationally expensive

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Practical Issues

- **discrete quantities**
  - directly usable
  - marginalization computationally expensive

- **continuous quantities**
  - approximations not in any reasonable class!
  - find optimal pdf $f$ in a predefined class $\mathcal{F}$
  - we have an algorithm for $\mathcal{F}$ being class of mixtures
Conclusions

The proposed method

- fulfills our requirements on ideal pdf fusion
  - independence on the ordering of sources
  - feasible for both discrete and continuous quantities
- fits well into other technologies in our framework
- will be implemented in Matlab toolbox MIXTOOLS 3000