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Image Fusion via Multichannel Blind Deconvolution

Ph.D. Thesis

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Abstract

The thesis focuses on fusion of degraded images originating from one source with the aim of obtaining an undegraded image of the source. Depending on the type of the degradation, a formalized system of the most common fusion problems is built. The unknown degradation we deal with is additive noise and space-invariant blurs modeled by convolution. The fusion process is then referred to as multichannel blind deconvolution and it frequently occurs in microscopy imaging, remote sensing, astronomical imaging, etc.

A novel iterative algorithm is proposed, which solves an energy minimization problem by means of an alternating minimization scheme. The energy functional, which is utilized here, incorporates regularization of the original image and blurs. Anisotropic regularization of the image based on total variation and the Mumford-Shah functional is implemented. Regularization of the blurs emanates from the multichannel framework of the problem, in particular, from mutual relations between channels degraded with different blurs. A better restoration performance was achieved in comparison with previously proposed multichannel blind approaches. Primarily, an enhanced noise robustness was observed. An accurate estimation of the blur size is however necessary.

The same restoration problem is revisited and a maximum a posterior estimate is derived. A priori probabilities of the original image and blurs are approximated by total variation semi-norm and by between-channel relations, respectively. This stochastic approach handles correctly situations when the blur size is overestimated. Moreover, translation misregistration of the channels up to a certain extent can be automatically removed in the restoration process.

Capabilities of the proposed methods are illustrated not only on synthetic data but also on real data acquired with a digital camera and on astronomical data.

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Contents

Chapter 1

Introduction

Our society is often called an "information society". It could also be referred to as an "*image society*". This is not only because the image is a very powerful and widely used medium of communication, but also because it is an easy, compact, and readily available way in which to represent the world that surrounds us. It is surprising indeed how omnipresent images are in our everyday lives.

Recent advances in acquisition devices are one of the main reasons responsible for such a phenomenon. Another reason is the increase in the capacity of computers, which enables us to process more and more data. Digital data are nowadays easy to acquire, store and efficiently process. This has given rise to new disciplines generally known as *image processing*, *image analysis* and *computer vision*; these terms differ in what kind of output information is required. In image processing, image intensity values are used to generate an image or images in a certain sense improved. Image analysis deals mainly with intensity values, which are often enriched with additional information that helps to construct a symbolic description of the content of the image. Computer (or machine) vision is mostly concerned with the 2-D representation of a 3-D world; it performs some sort of abstract reasoning followed by decision-making and action. These disciplines have found practical applications in several unfolding fields.

Medical imagery is one of the fields that has made use of images since the earliest days. Many devices based on ultrasound, X-rays, magnetic resonance, scanners, etc. produce images that are subjected to various processing tasks, such as quality improvement, feature enhancement and extraction, or integration of different pieces of information (*fusion*).

Probably the first extreme need for an ability to retrieve meaningful information from degraded images was encountered in *astronomical imaging*. Many space endeavors utilize state-of-the-art image processing tools. Ground-based telescopes and extraterrestrial observations of the universe provide numerous data that are prime examples of the need for quality improvement and feature enhancement.

Another important field that directly concerns us is *remote sensing*. This consists of applications, in which we analyze, measure, or interpret scenes at a distance. Apart from defense and video surveillance applications and road traffic analysis, the observation of earth resources is another important field. Image processing can aid in the tracking and quantifying changes in forests, water supplies, pollution, etc. It can also be used for weather forecasting.

During the last decade, *video processing* has become a prime target of investigation. This is because in many applications we need to process not only still images but also sequences of images. Typical examples are weather forecasting, video compression, movement tracking, segmentation, and restoration of old movies.

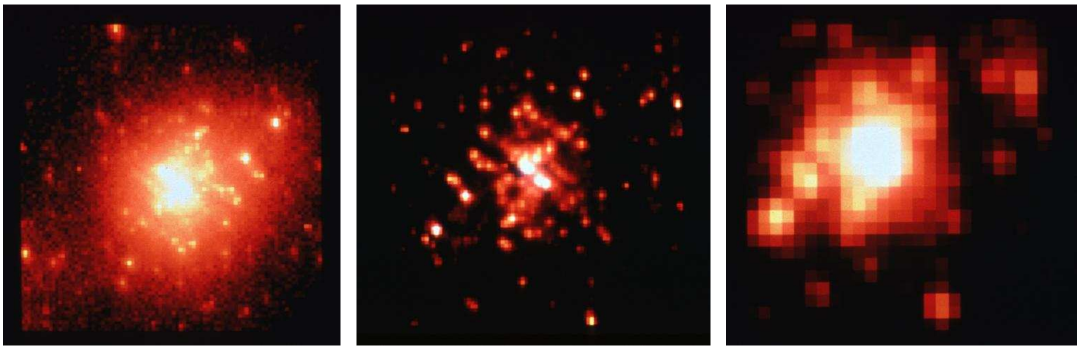
A somewhat less related field that is beginning to take advantage of the most advanced processing tools is *art conservation*, as well as art in general. Many attempts have been initiated to create complex databases for the organization of art archives with "smart" query capabilities based on some degree of image cognition. Other tasks that can benefit from image processing include automatic determination of authorship, reconstruction of prehistoric art and validation of conservation efforts.

1.1 Motivation

A corner stone of image processing and analysis that is rarely omitted from most real application tasks is *image restoration*. **The aim of the restoration process is to recover an accurate representation of a given scene from degraded images acquired with a imaging system.**

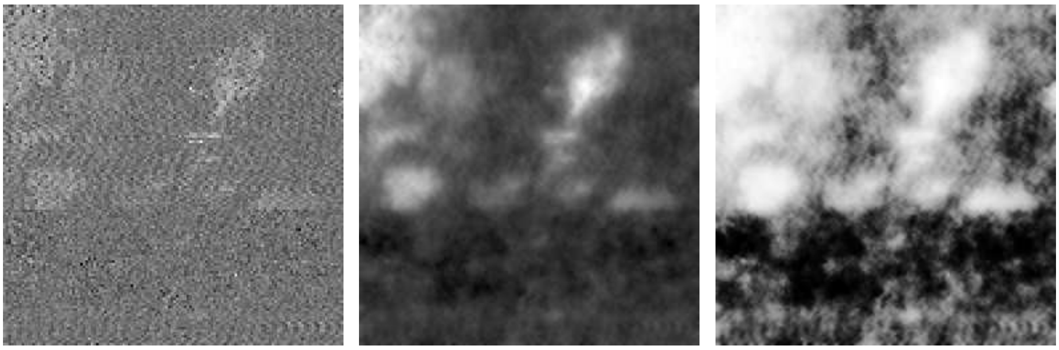
There are many sources of corruption or distortion that we have to cope with. Light rays (or other types of electromagnetic waves) reflected by objects on the scene travel to measuring sensors through a transport medium, e.g., the atmosphere. Inevitably, each transport medium modifies the signal in some way. The imaging system is thus subject to blurring due to the rapidly changing index of refraction of the medium, the finite broadcast bandwidth and/or the object motion. The source of corruption and its characteristics are often difficult to predict and can cause a restoration malfunction. In addition, the signal is corrupted inside a focusing set after reaching the sensor. This degradation is inherent to the system and cannot be bypassed, but it can often be measured and accounted for; typical examples are all sorts of lens imperfections. Finally, the signal must be stored on photographic material or first digitized with CCD's and then stored. In both cases the recording exhibits a number of degradations. Digital imaging systems suffer from low resolution and low sensitivity to the input signal, which are imposed by a finite number of intensity levels and a finite storage capacity. In analog systems, resolution artifacts are caused by the limited size of photographic material grain. A good overview of digital image restoration techniques can be found in [1]. A well-known example of successful image restoration was given for images taken by the Hubble Space Telescope (HST) before the corrective optics were installed in 1993. Due to the optical aberration in HST's primary mirror, starlight was blurred and the telescope's ability to see faint structure was limited. Fig. 1.1(a) illustrates a cluster of tightly-packed young stars captured by HST. The undesirable fuzzy halos around the stars are substantially reduced by computer restoration as shown in Fig. 1.1(b). In comparison in Fig. 1.1(c), the same cluster measured by a ground-based telescope demonstrates the undesirable impact of the atmosphere that is not present in HST's images and which makes HST so praised.

Noise is another crucial factor that severely affects the quality of image acquisition. In all real applications, measurements are degraded by noise. By utilizing suitable measuring techniques and appropriate devices, it can be considerably diminished but unfortunately never canceled. In the course of acquiring, transmitting, or processing a digital image, the noise-induced degradation may be dependent or independent of data. We identify several noise models that are characterized by different probability distributions. The most common is certainly Gaussian noise. Some applications, however, exhibit more specific ones, like the speckle noise in radar images and Poisson noise in tomography or astronomy.



(a) HST's observation (b) Image restoration (c) Ground-based observation

Figure 1.1: Hubble Space Telescope: View of a remarkable cluster of tightly-packed young stars 160,000 light years from Earth in the Large Magellanic Cloud Galaxy. (a) raw image as captured by HST exhibiting severe effects of mirror aberration; (b) computer image restoration with fuzzy halos removed; (c) the same cluster observed from a ground-based telescope. The images are credited to STScl and NASA.

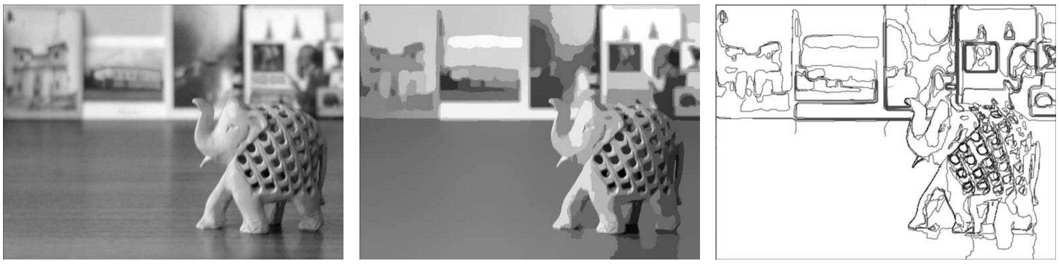


(a) BEEM noisy image (b) Denoising (c) Denoising and hist. equal.

Figure 1.2: Image denoising: Ballistic Electron Emission Microscopy is prone to heavy noise. (a) shows the raw BEEM image, (b) is the image reconstructed by a nonlinear filter that preserves fine details and (c) shows results of successive histogram equalization.

Other noise models are Laplacian and impulsive. An overview of standard denoising techniques can be found in [2] and for more advanced non-linear methods, refer to [3] and [4]. Fig. 1.2 presents a noisy image taken by a ballistic electron emission microscope (BEEM) and two results of restoration by nonlinear filtering and histogram equalization.

A problem that is more specific to image analysis is *segmentation*. **To segment an image means to partition the domain of its definition into several regions on which the image is homogeneous and there are abrupt changes between regions.** The exact meaning of "homogeneous" depends on the application. Usually it means a behavior according to some a priori probability distribution, which could be constant intensity or uniform image texture. Segmentation problems arise in many tasks for automatic target recognition. Some examples are the identification of tree-grass boundaries in synthetic aperture radar images, the localization of hot targets in infrared radar and the separation of foreground and background in laser radar. In Fig. 1.3, an example of 3-D scene segmentation is illustrated. Medical imaging provides many data that benefit from segmentation



(a) (b) (c)

Figure 1.3: Image segmentation: (a) picture taken by a standard digital camera, (b) segmentation using the Mumford-Shah functional, (c) level lines of the segmented image

as well. Images resulting from ultrasound, magnetic resonance imaging, tomography, dermatoscopy contain certain objects of interest, e.g., an internal organ, a tumor, or a boundary between the gray and white matter, that are necessary to localize and extract. The main challenges of the segmentation problem depend on the object and on the imaging modality. Image segmentation is closely related to restoration. Indeed, the solution to one problem makes the other easier. If the region boundaries are known, recovery of the uncorrupted image is simpler, and vice versa, segmentation is easier once a good estimate of the image is at hand. It is therefore natural that many segmentation algorithms are closely related to restoration techniques. In fact, some methods combine both tasks and produce the edge locations and image intensity values simultaneously.

The efficiency of restoration and segmentation methods depends heavily on the amount of a priori knowledge of the imaging system. This designates wide variety of aspects. The fundamental one is definitely the understanding of the acquisition process, i.e., the prior knowledge or assumptions about the type of degradation. Another useful information, but often difficult to obtain, is the characteristic of measured objects.

Intuitively, one may expect that the usage of different sensors or repeated measurement under varying conditions also provide extra information that improves restoration. This brings us to a new field called data fusion. It means an approach to information extraction spontaneously adopted in several domains. An illustration is given by the human system which calls upon its different senses, its memory and its reasoning capabilities to perform deductions from the information it perceives. Data fusion encompasses a very wide domain. In a broad sense, the definition could be as follows. **Data fusion is a set of methods, tools and means for the alliance of data originating from various sources of different nature in order to obtain information whose quality cannot be achieved otherwise.**

Since we concern ourselves with *image fusion*, all input information about the scene is in the form of digital images. The goal of image fusion is to integrate complementary multisensor, multiview or multitemporal information into one new image which is in some sense more suitable for further analysis. Image fusion has been used in many application areas. In remote sensing and in astronomy [5, 6], multisensor fusion is used to achieve high spatial and spectral resolutions by combining images from two sensors, one of which has high spatial resolution and the other one high spectral resolution. Numerous applications emanate from medical imagery like simultaneous evaluation of CT (computer tomography), NMR (nuclear magnetic resonance) and/or PET (positron emission tomography) images. In the case of multiview fusion, a set of images of the same scene taken by the same sensor but from different viewpoints is fused to obtain an image with higher resolution than the sensor normally provides or to recover the 3-D representation of the scene (shape from stereo). The multitemporal approach recognizes two different aims. Images of the same scene are acquired at different time instances either to find and evaluate changes in the scene or to obtain a less degraded image of the scene. The former aim is common in medical imaging, especially in change detection of organs and tumors, and in remote sensing for monitoring land or forest exploitation. The acquisition period is usually months or years. The latter aim requires the different measurements to be much closer to each other, typically in the scale of seconds, and possibly under different conditions. This setting occurs, e.g., in astronomy, where objects are observed under different atmospheric conditions or at different aperture.

It was mentioned above that image fusion is a powerful and frequent tool in medical imaging (see [7] or [8] for instance), astronomy and remote sensing. Image fusion is also often used in machine vision [9], [10] and mobile robot navigation [11], in scene understanding [12], [13], [14], in boundary tracking [15], in object and target recognition [16], [17], in traffic control [18] and in automatic change detection and monitoring of dynamic processes [19].

The main topic of this thesis is fusion towards image restoration, i.e., images are assumed to be degraded and represent the same modality of the scene. In this particular case, the input degraded images are called channels and we speak about multichannel restoration, which more accurately reflects the theme under investigation.

Chapter 2

State of the art

In this chapter, we discuss in detail the problem of monomodal fusion from the perspective of the above discussion. The basic terminology used in this field is introduced and then the state of the art is given.

2.1 Basic framework

We deal here with *monomodal fusion of degrade images*. The word monomodal states that the degrade images have one common image source and thus do not represent different physical properties of a observed scene. Monomodal fusion can be in general formulated as follows.

Let $u(x, y)$ be an ideal, perfect image of the scene and let $z_1(x, y), \cdots, z_n(x, y)$ be images of the same scene obtained from different sensors measuring the same physical character, from different viewpoints, at different time instances and/or under different observational conditions. The relation between each z_i and u is expressed as

$$z_i(x, y) = \mathcal{D}_i(u(x, y)) \,, \tag{2.1}$$

where $\mathcal{D}_i$ is an operator describing all kinds of image degradations including imaging geometry, blur, noise and other factors caused by the acquisition process. The major goal of the fusion is to obtain an image $\hat{u}$ as a "good estimate" of u, that means $\hat{u}$ should be in some sense better representation of the original scene than each individual frame z_i is.

The fusion methodology depends significantly on the type of the degradation operators $\mathcal{D}_i$. In the sequel, the degradations are assumed to be compositions of image blurring, additive noise and geometric deformations caused by imaging geometry. The blur is introduced into the image by such factors as diffraction, lens aberration, motion of the scene, wrong focus and medium turbulence. Under these assumptions, (2.1) has the following form:

$$z_i(\tau_i(x, y)) = H_i(u(x, y)) + n_i(x, y), \tag{2.2}$$

where H_i is a linear bounded operator of the i-th imaging system that performs the intensity value deformations, $n_i(x, y)$ is additive noise and τ_i is a transform of spatial coordinates. Operator H_i and function τ_i are assumed to be unknown. In this form, the problem is too general to be solvable. However, in many practical cases H_i can be successfully modeled as an integral transform with kernel h_i. We thus write

$$H_i(u(x, y)) = \int h_i(x, y; s, t) u(x - s, y - t) ds dt \,, \tag{2.3}$$

where $h_i(x, y; s, t)$ is the impulse response or point spread function (PSF) at location (x, y). In this case, h_i varies within the image domain and we speak about a *space-variant* imaging system. If h_i is not dependent on (x, y), i.e. $h_i(x, y; s, t) = h_i(s, t)$, we speak about a *space-invariant* imaging system and (2.3) takes the form of convolution $H_i(u) = h_i * u$.

Depending on the type of the degradation $\mathcal{D}$, we can build a formalized system of the most common fusion problems. We distinguish 4 models: *locally ideal imaging*, *uniformly blurred channels*, *misregistered uniformly blurred channels* and *realistic imaging*. Description of each model follows.

2.2 Locally ideal imaging

This is the simplest model that assumes space-variant PSF's in each channel. Every location (x, y) in the image is assumed to be acquired undistorted in (at least) one channel, which means that the corresponding local PSF is the delta function. No geometric deformations are assumed.

This model is applicable when we photograph a static scene with a known piecewise constant depth and we focus channel-by-channel on each depth level. Fig 2.1 illustrates this simple fusion task. Image fusion then consists of comparing the channels in the image domain [20, 21] or in the wavelet domain [22, 23], selecting the channel in which the pixel (or block) is depicted undistorted and, finally, of mosaicing the undistorted parts. To find the undistorted channel for the given pixel, local focus measure is calculated over its neighborhood and the channel which maximizes the focus measure is chosen. Various focus measures can be used for this purpose. Most of them are based on the idea to emphasize high frequencies of the image and measure their quantity. It corresponds with our intuitive expectation that the blurring suppresses high frequencies regardless of the particular PSF. Image variance [24], energy of a Fourier spectrum [25], norm of image gradient [24], norm of image Laplacian [24], image moments [26], and energy of high-pass bands of the wavelet transform [22, 23, 27] belong to the most popular focus measures. Fusion in the image domain is seriously affected by the size of the neighborhood on which the focus measure is calculated. On the other hand, fusion in the wavelet domain is very sensitive to translation changes in the source images.

2.3 Uniformly blurred channels

This model assumes "traditional" space-invariant convolution in each channel with no geometric deformations. The model describes for instance photographing a flat static scene with different (but always wrong) focus. The image fusion is performed via multichannel blind deconvolution. An example of blind deconvolution of sunspot images taken by a ground-based telescope is depicted in Fig. 2.2.

Blind deconvolution in its most general form is an inextricable problem. All the methods proposed in the literature inevitably make some assumptions about the PSF's h_i and/or the original image u. The deconvolution methods thus differ in the way what prior knowledge of the blurs and the original image is used and how it is implemented. This is a bit simplified but true statement. There are two basic approaches to multichannel deconvolution. The first one is to treat each channel separately by any singlechannel deconvolu-

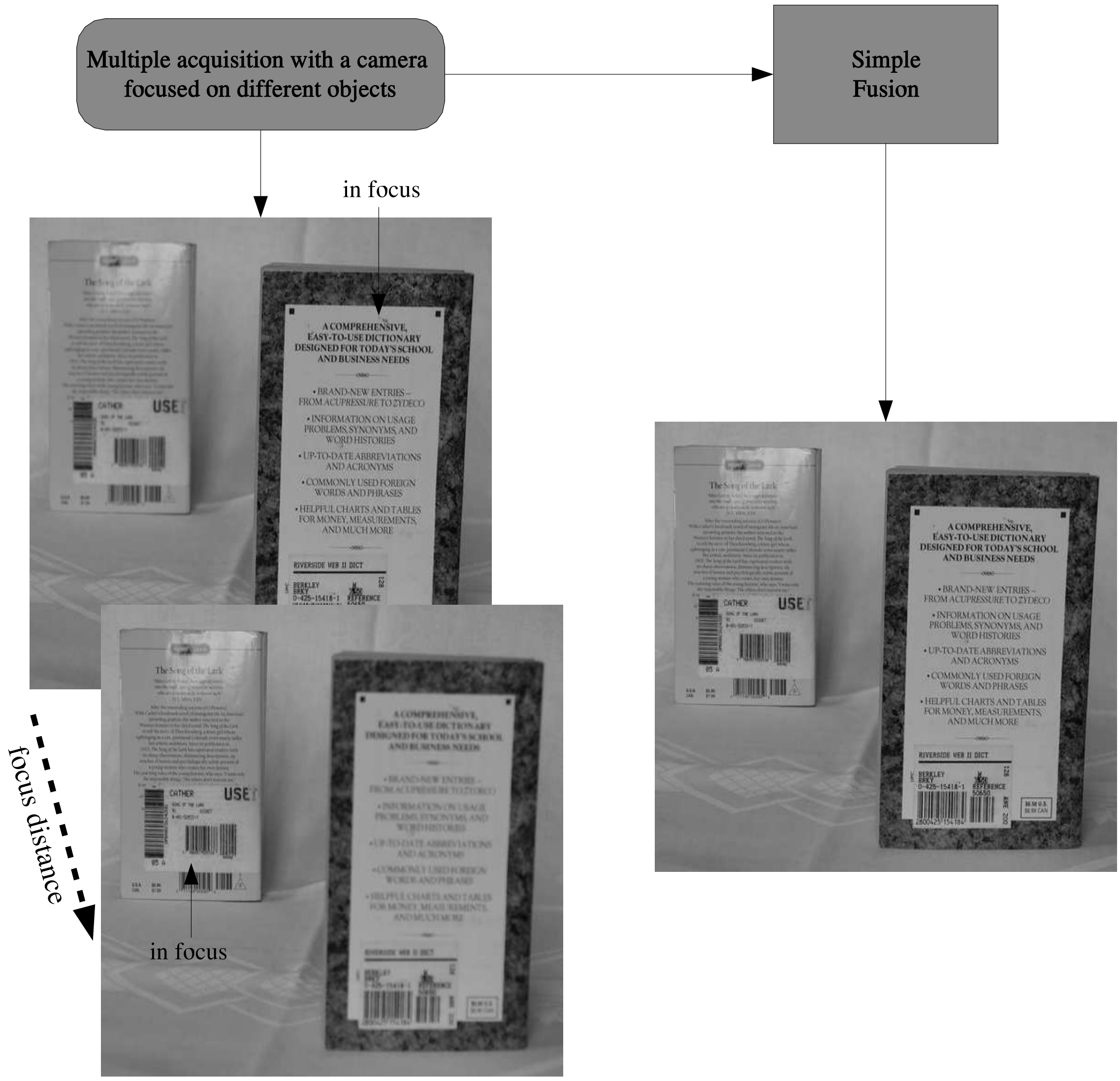


Figure 2.1: Locally ideal imaging: A camera with a limited depth of field can be set to different focus distances, and by mosaicing the acquired images, the whole 3-D scene is reconstructed.

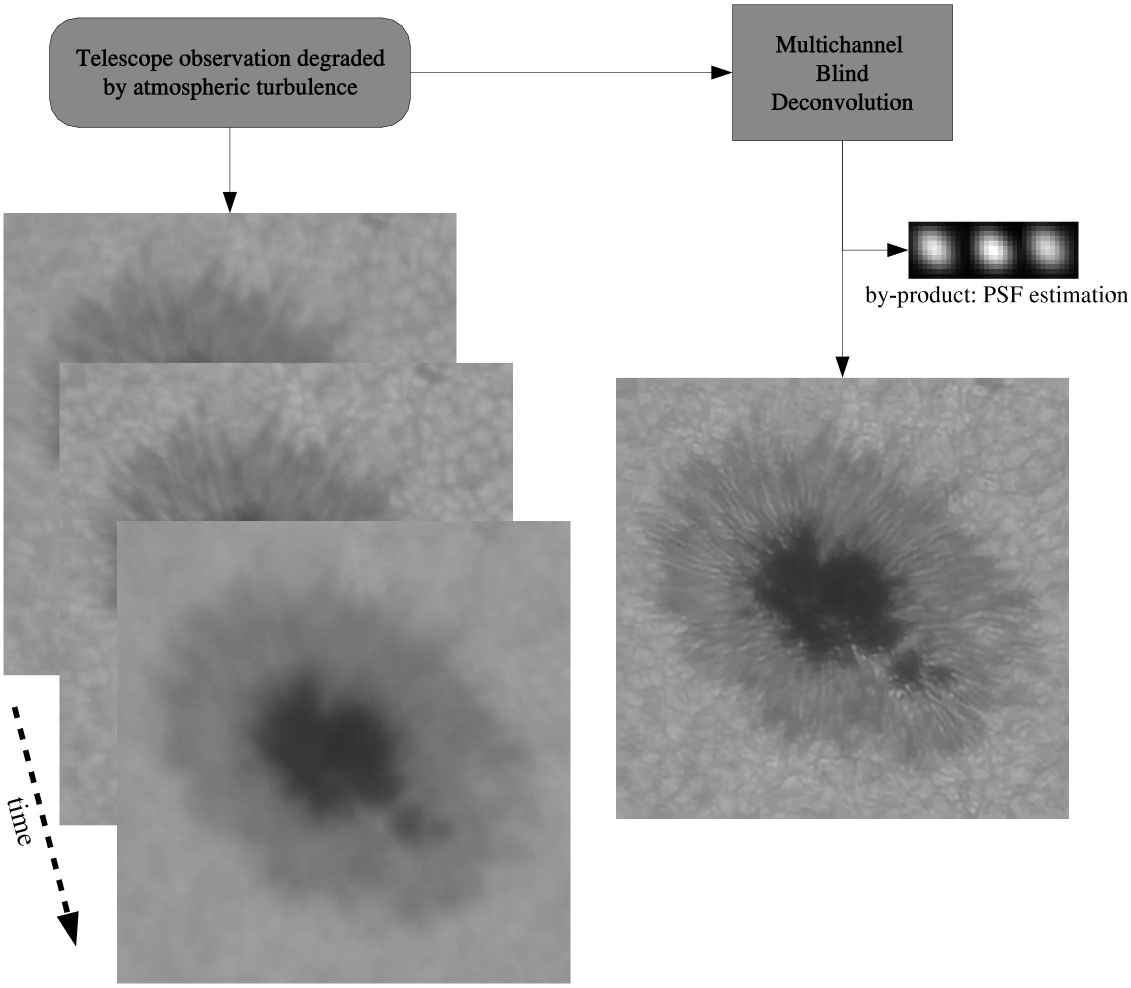


Figure 2.2: Uniformly blurred channels: Short-exposure images taken by a ground-based telescope are blurred primarily due to perturbations of wavefronts in the Earth atmosphere. The perturbations vary in time, which means that different blurs occure in a sequence of measurements but the blur in each measurement is spatially uniform. Three pictures of a sunspot taken shortly one after another and their restoration is depicited here. No misalignment of images is present.

tion method and then to combine the results, the other one is to employ a deconvolution method that is multichannel in its nature.

Numerous singlechannel blind deconvolution methods have been published extensively in the literature in last two decades (see [28] or [29] for a basic survey). Blind deconvolution was shown to be an ill-posed problem, which does not have a unique solution and the computational complexity of which could be extremely high.

There are several major groups of blind deconvolution methods. Well-known *parametric methods* assume that a parametric model of the PSF is given a priori. Investigating the zero patterns of the Fourier transform or of the cepstrum of $z(x, y)$, the unknown parameters are estimated [30, 31, 32, 33]. This approach is very powerful in motion and out-of-focus deblurring, for instance. Parametric methods for estimating more general motion blurs by means of statistical moments of the motion functions [34] and by means of autocorrelation of directional derivatives [35] were also proposed. Another blur identification method [36] selects from an admissible collection of PSF's a candidate with the restoration residual power spectrum that best matches the expected residual power spectrum.

Promising results were achieved by a *zero-sheet separation* method, which was first introduced in [37] and further developed in [38, 39]. The method is based on the analytical properties of the z-transform in 2-D. It was proven that the zeros of the z-transform of each $u(x, y)$ and $h(x, y)$ lie on distinct continuous surfaces called zero sheets. Separating these two zero sheets from each other, we can restore both $u(x, y)$ and $h(x, y)$ up to a scaling factor. Although conceptually the zero-sheet method is correct, it has little practical application since the algorithm is highly sensitive to noise and prone to numerical inaccuracy for large image sizes.

Another group of methods is based on the modeling the image by a stochastic process. The original image is modeled as an autoregressive (AR) process and the blur as a moving average (MA) process. The blurred image is then a result of a mixed autoregressive moving average (ARMA) process and the MA process identified by this model is considered as a description of the PSF. In this way the problem of the PSF estimation is transformed into the problem of determining parameters of the ARMA model. The methods of this category differ in how the ARMA parameters are estimated. Basic approaches are *maximum likelihood estimation* [40] with an expectation-maximization algorithm [41] or without [42], their multichannel extensions [43, 44] and *generalized cross-validation* [45].

A *projection-based* approach to blind deconvolution proposed in [46] attempts to incorporate prior knowledge about the original image and the PSF through constraint sets. This method was proven to perform well even if the prior information was not perfect. However, the solution may be not unique. Besides the projection-based method, there is a wide variety of other non-parametric algorithms which use a priori deterministic or stochastic constraints, such as image positivity, the size of the PSF support, expected power spectral densities of the PSF and the original image, etc. One of the oldest is *iterative blind deconvolution* [47] with its several enhancements [48, 49, 50, 51]. It is based on iterative Wiener restoration that alternates between h and u and, in each iteration, the constraints are imposed on partial results. This method is robust to noise but suffers from uncertain convergent properties. Deconvolution via *simulated annealing* [52] fall into this category as well. Very promising results have been achieved with a *nonnegativity and support constraints recursive inverse filtering* (NAS-RIF) algorithm proposed in [53] and extended in [54, 55]. However, this method is applicable to images that contain objects of finite support on a uniform background and the object support must be determined in advance. Similar inverse filtering that admits solutions from a convex feasible set of filters was proposed in [56]. Stochastic non-parametric approaches based on the Bayes theorem include maximum a posteriori (MAP) estimation [57] and a Richardson-Lucy multichannel algorithm [58] which is efficient for a Poisson noise model. A group of deconvolution methods based on *higher-order statistics* (HOS) was designed particularly for restoring images with textures [59, 60, 61].

Restoration methods that have evolved from minimization of *variational integrals* form a very interesting branch [62, 63, 64]. The variational integrals, like e.g. total variation (TV), play a prominent role in image denoising due to their anisotropic diffusion ability which preserves edges in images. The main advantage is the sharpness of restored images but the convergence properties are somewhat dubious [65].

All the methods discussed so far have not fully exploited the potential of the multichannel framework. The development of intrinsically multichannel methods has begun just recently. There are three main advantages of the multichannel approach in comparison with the singlechannel one: noise reduction through statistical independence between image noise fields, potential elimination of zeros in the denominator of the restoration filter, and mainly, higher amount of information about the original image when the PSF's are diverse enough. The lack of information from one blur in one frequency is supplemented by the information at the same frequency from others.

One of the earliest methods [66] was designed particularly for images blurred by atmospheric turbulence. Harikumar et al. [67, 68] proposed two indirect algorithms, which first estimate the PSF's and then recover the original image by a standard nonblind method. The first one is a *subspace technique* and the PSF's are equal to the minimum eigenvector of a special matrix constructed only by the blurred images. The second one solves the subspace problem as a *maximum likelihood* estimator. Necessary assumptions for perfect recovery of the blurs are exact knowledge of the blur size, noise-free environment and channel coprimeness, i.e. a scalar constant is the only common factor of the blurs. Giannakis et al. [69, 70] (and at the same time Harikumar et al. [71]) developed another indirect algorithm based on Bezout's identity of coprime polynomials which finds *inverse filters* and by convolving the filters with the observed images recovers the original image. Both, the subspace method and inverse filters, are vulnerable to noise and even for a moderate noise level restoration may break down. In the latter case, noise amplification can be attenuated to a certain extent by increasing the inverse filter order, which comes at the expense of deblurring. Pillai et al. [72] have proposed another intrinsically multichannel method based on the *greatest common divisor* which is, unfortunately, even less numerically stable than the previous ones. Pai et al. [73, 74] came with two direct multichannel restoration algorithms that, contrary to Harikumar's and Giannakis' indirect ones, estimate directly the original image from the *null space* or from the *range* of a special matrix. In noisy cases, the direct algorithms are more stable than the indirect ones. Nevertheless, all the algorithms lack the necessary robustness since they do not include any noise assumptions in their derivation and omit regularization terms.

2.4 Misregistered uniformly blurred channels

This is a generalization of the previous model which allows rigid-body geometric differences (misregistrations) between the channels, i.e., τ_i is a rotation and translation of the channel. This model is applicable in numerous practical tasks when the scene or the camera moves between consecutive channel acquisition. To fuse images degraded according to this model, image registration must precede multichannel blind deconvolution.

Image registration in general is a process of transforming two or more images into geometrically equivalent form. It eliminates the degradation effects caused by geometric distortion. From mathematical point of view, it consists of approximating τ_i^{-1} and of resampling the image. For images which are not blurred, the registration has been extensively studied in the recent literature (see [75], [76] and [77] for a survey). However, blurred images require special registration techniques. They can be, as well as the general-purpose registration methods, divided in two groups – global and landmark-based ones. Regardless of the particular technique, all feature extraction methods, similarity measures, and matching algorithms used in the registration process must be insensitive to image blurring.

Global methods do not search for particular landmarks in the images. They try to estimate directly the between-channel translation and rotation. Myles and Lobo [78] proposed an iterative method working well if a good initial estimate of the transformation parameters is available. Zhang et al. [79], [80] proposed to estimate the registration parameters by bringing the channels into canonical form. Since blur-invariant moments were used to define the normalization constraints, neither the type nor the level of the blur influences the parameter estimation. Kubota et al. [81] proposed a two-stage registration method based on hierarchical matching, where the amount of blur is considered as another parameter of the search space. Zhang and Blum [82] proposed an iterative multiscale registration based on optical flow estimation in each scale, claiming that optical flow estimation is robust to image blurring. All global methods require considerable (or even complete) spatial overlap of the channels to yield reliable results, which is their major drawback.

Landmark-based blur-invariant registration methods have appeared very recently, just after the first paper on the moment-based blur-invariant features [83]. Originally, these features could only be used for registration of mutually shifted images [84], [85]. The proposal of their rotational-invariant version [86] in combination with a robust detector of salient points [87] led to the registration methods that are able to handle blurred, shifted and rotated images [88], [89].

Theoretically, after the images are registered, multichannel blind deconvolution from the previous model could be employed. However, registration is always prone to small inaccuracies and current blind deconvolution methods are not robust enough to deal with it. This situation occurs, e.g., when a vibrating object is photographed. Due to an irregular camera-object motion, acquired images are not only degraded with random motion blurs but also mutually translated; see Fig. 2.3.

2.5 Realistic imaging

This model comprises space-variant blurring as well as nonrigid geometric differences between the channels, i.e., τ_i is affine or projective coordinate transform. Almost all sit-

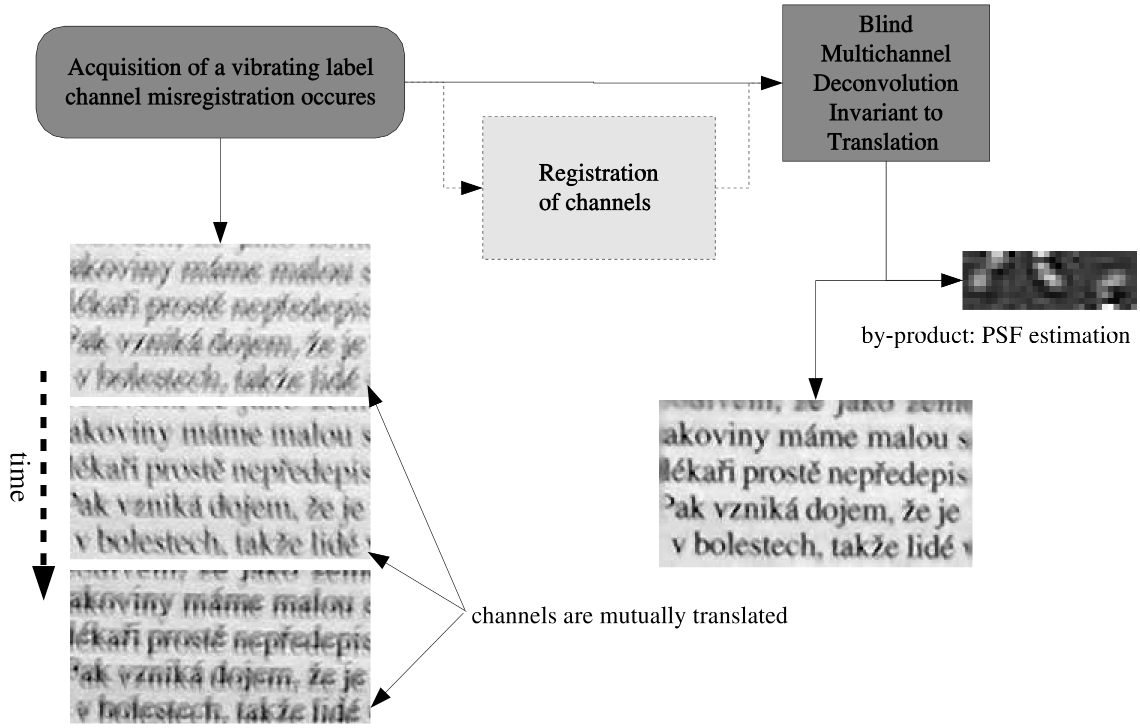


Figure 2.3: Misregistered uniformly blurred channels: When multiple acquisition is performed, individual images are in general not spatially aligned. After channel registration, small translational misalignments are still frequently observed. The deconvolution method proposed in the thesis is robust enough and can be thus applied to this case.

uations occurring in practice can be described by this model with a reasonable accuracy. Like in the previous model, the image fusion consists of image registration and multichannel blind deconvolution but there is a significant difference. Here, the registration technique must be able to remove nonrigid deformations and the deconvolution must be space-variant. This model is not a simple extension of the previous models. It requires qualitatively new approaches and methods; see Fig. 2.4 for illustration.

Shift-variant blind restoration is qualitatively far more difficult than the shift-invariant problems considered so far and not many attempts have been made in past to solve this problem. To our knowledge, there is a shift-variant extension of the expectation-maximization algorithm [90] that divides the image into stationary regions, restoration by anisotropic regularization [64] for the parametrized PSF and a multiscale attempt [91]. A successful method would certainly be praised, since many common imaging systems exhibit space-variable PSF's; just consider, for example, a picture of a 3-D scene taken by a camera with a shallow depth of field compare to the depth of the scene.

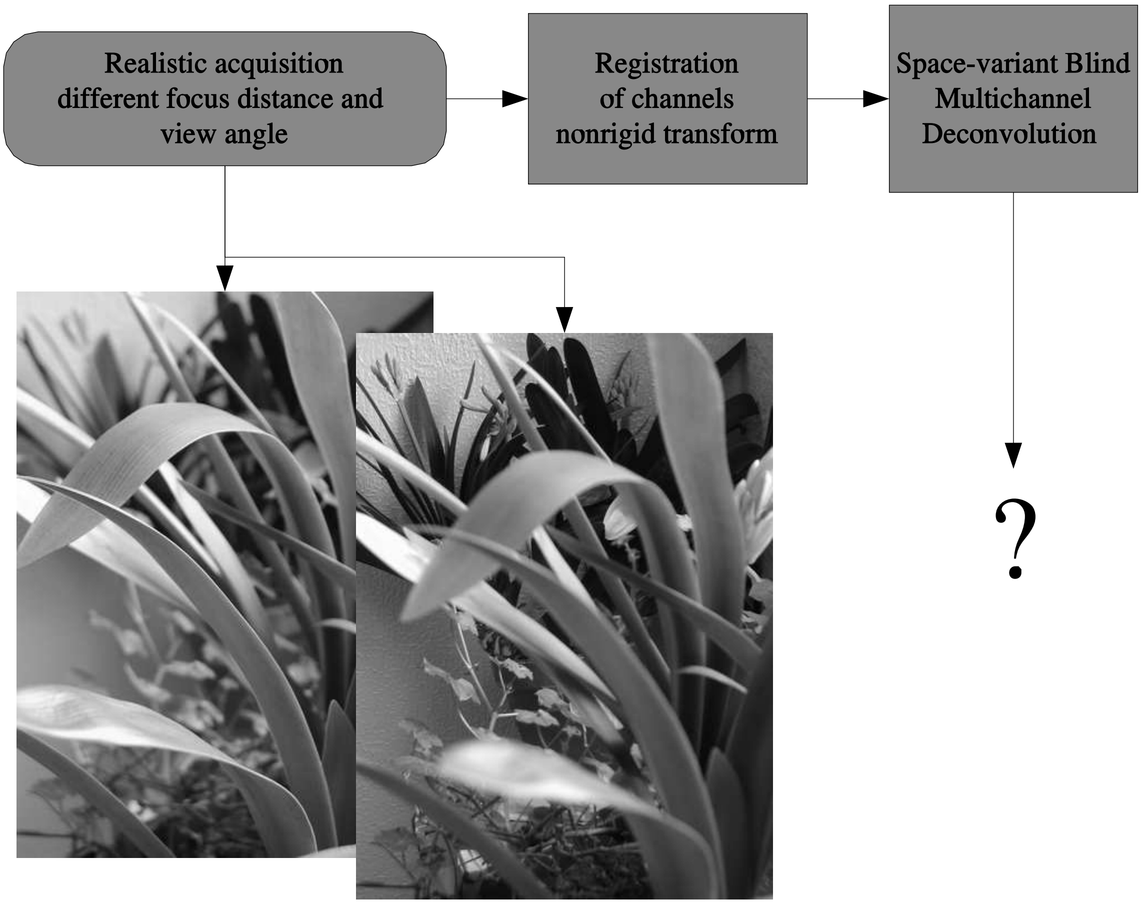


Figure 2.4: Realistic imaging: The most general model, which assumes nonrigid transformations between different acquisitions and the shift-variant convolution model. Taking a picture of a 3-D scene from different view angles is a typical example.

Chapter 3

Objectives

In this chapter, the problem that is analyzed in the thesis is formulated and set in to the frame of the previous discussion. The objectives of the thesis are clearly stated as well.

3.1 Problem formulation

The overview in Chapter 2 clearly illustrates that multichannel blind deconvolution (monomodal fusion) occurs in real applications in different forms and that a wide variety of techniques some more and some less application-dependent were proposed in the literature to solve it. The first three models 2.2, 2.3 and 2.4 are mathematically tractable and form the primary topic of current research in this field. The last model 2.5 (realistic imaging) with nonrigid misregistration and shift-variant convolution is far more challenging but beyond the scope of this work. The few attempts that try to solve the realistic model are still in their initial stage and do not provide a thorough framework. However, the realistic imaging will certainly be a subject of research in the near future.

In the sequel, we are concerned with blind multichannel deconvolution, therefore, model 2.3 (uniformly blurred channels) and model 2.4 (misregistered uniformly blurred channels) form the domain of our investigation. The singlechannel methods are of not much interest, since they blatantly neglect the complementary information provided by multiple channels. On the other hand, the intrinsically multichannel methods proposed so far combine seamlessly the information from multiple channels but exhibit some unpleasant characteristics. Mainly, it is the poor performance under noisy conditions and the requirement of exact knowledge of the blur size. Both items hinder current multichannel methods from being used in real applications.

To increase the noise robustness, a natural approach would be to incorporate noise assumptions into the intrinsically multichannel methods. This can be done, for example, by extending standard denoising techniques into the multichannel framework so that mutual relations between channels are preserved during the denoising and deconvolution procedure.

The requirement of exact knowledge of the blur size is very restrictive and often difficult to satisfy for real images. At the present, this problem is addressed by using blur-size estimation techniques prior to the deconvolution process. The estimation techniques proposed for this purpose are however prone to error and they perform even less satisfactory as noise level increases. More reliable, but also more computationally demanding, approach is to implement a full search, i.e., restoration is repeated for the different blur sizes and the best result is retained. It is therefore clear that a deconvolution method, which does not require the precise size of blurs, could be applied to a much broader range of imaging problems. Moreover, one can intuitively deduce that such a method can cope intrinsically with simple between-channel misregistration like translation, since by shifting the centers of the blurs the translation transform is performed during deconvolution.

The mathematical model that is considered in the thesis takes the following form:

$$z_i(x + t_i^x, y + t_i^y) = (h_i * u)(x, y) + n_i(x, y)\,, \tag{3.1}$$

where z_i, h_i, n_i, u are the i-th degraded image, PSF, noise, original image, respectively, (t_i^x, t_i^y) is the translation vector in the i-th channel and $*$ denotes 2-D convolution.

3.2 Thesis objectives

The main goal of the thesis can be formulated as follows:

Propose a multichannel blind restoration method, which performs satisfactory on images severely degraded by noise and which is robust to spatial misalignment of the channels. Demonstrate the capability of the proposed method on data obtained from real applications, primarily astronomical imaging.

Chapter 4

Results

It was briefly outlined in the previous chapters, how to meet the objectives of the thesis and what are the practical requirements that justify the goals declared.

The first objective, noise robustness, is addressed in a simple but very effective manner. The straightforward approach adopted here is to incorporate mutual relations between channels into a denoising algorithm. The mutual relation between channels forms the fundamental notion of the intrinsically multichannel deconvolution methods and it is based on the assumption that the channels are sufficiently diverse, more precisely, that they are weakly coprime, i.e., the z-transforms of PSF's have no common factor. Although coprimeness may seem to be rather theoretical and difficult to satisfy in real cases, the opposite is true. The necessary channel disparity is mostly guaranteed by the nature of acquisition schemes and random processes therein.

A wide range of noise removal approaches has been proposed in the literature. In general we distinguish two main branches: linear and nonlinear. Linear denoising techniques are easy to implement but they tend to blur images and their role in deconvolution is thus limited. The key role plays the heat equation[1] ($\frac{\partial u}{\partial t} = \Delta u$) that is proved to be an asymptotic state of any iterative linear isotropic smoothing and which is the reason why linear filtering has the overall uniform smoothing effect. Much more encouraging results are achieved via nonlinear filtering. The basic idea is to let the smoothing happen only in areas that are relatively flat, which implies that image edges and fine details are preserved. This type of selective smoothing can be regarded as anisotropic diffusion. A simple example is diffusion that occurs solely in the direction orthogonal to the gradient, i.e., $\frac{\partial u}{\partial t} = div(Du/|Du|)$, where Du is the gradient of u in the distributional sense. They are the partial differential equation (PDE) and its viscosity solution that became the tools for tackling such nonlinear filtering problems. Many PDE's come from variational principles of the form $\int F(|Du|)$, where F is a smooth functional. The link between these two approaches is that the gradient descent solution of a minimization problem $\min_u \int F(|Du|)$ is associated with a PDE $\frac{\partial u}{\partial t} = div(g(|Du|)Du)$, where $g(s) = F'(s)/s$. In the case of multichannel deconvolution, the minimization problem (and likewise the PDE) is embedded with two distinct constraints. The first one measures the fidelity to the degraded images based on our model (3.1). The second one has its origin in the mutual relation between channels that was mentioned above.

The drawback of the constrained minimization problem is the necessity to properly set weighting constants (or Lagrange multipliers) for the constraints. To partially alleviate

[1]The heat equation (or diffusion equation) is a partial differential equation that governs the standard diffusion processes in nature and its solution is convolution with a Gaussian kernel.

this problem, the minimization problem is revised here and formulated as a maximum a posteriori (MAP) estimation problem that is iteratively solved. This stochastic approach provides a different angle of view that reveals some surprising properties.

The MAP approach proposed here is not seriously dependent on the exact knowledge of the blur size. In other words, if the blur size is overestimated the original shape of the blurs can still be recovered, more or less. The validity of the solution and the convergence rate depends on the degree of overestimation. If the blur size is underestimated the restoration problem does not have a solution. This partial freedom in the blur size estimation equips the MAP algorithm with an ability to handle spatially misaligned channels and perform simple translation registration, which is the second objective of the thesis.

The thesis is a collection of four research papers:

- F. Šroubek and J. Flusser, "Multichannel Blind Iterative Image Restoration," *IEEE Trans. Image Processing*, accepted in March 2003 (see Section 4.1),
- F. Šroubek and J. Flusser, "Multichannel blind deconvolution of spatially misaligned images," *IEEE Trans. Image Processing*, submitted in March 2003 (see Section 4.2), a shorter version will appear in *Proc. of the 3rd Int'l Symposium on Image and Signal Processing and Analysis*, ISPA, 2003,
- F. Šroubek, J. Flusser, T. Suk, and S. Šimberová, "Multichannel Blind Deconvolution of the short-exposure astronomical images," in *Proc. of the 15th International Conference on Pattern Recognition*, pp. 53–56, 2000 (see Section 4.3),
- B. Zitová, J. Flusser, and F. Šroubek, "Application of Image Processing to The Medieval Mosaic Conservation," *Pattern Analysis and Applications*, submitted (see Section 4.4), a shorter version published in *Proc. of the IEEE International Conference on Image Processing,* pp. 993-996, ICIP'02, IEEE, Rochester 2002.

The first two journal papers propose two different approaches that give answers to the thesis objectives and demonstrate their applicability to restoration of images acquired in real situations, like astronomical imaging and digital camera photography.

The third in the collection illustrates multichannel restoration of blurred astronomical images.

The last paper is a noteworthy attempt to assess an art conservation task, which was conducted in the Department of Image Processing, ÚTIA, Prague. Different restoration techniques were implemented and evaluated for this particular task.

Although the papers were co-authored, the author of the thesis is the originator of the first three manuscripts and of the image restoration section in the fourth manuscript. My own contribution (estimated by all co-authors) is 90%, 90%, 50% and 30%, respectively. Extended abstracts of the papers together with a discussion concerning meeting the goals of the thesis are given in the following sections. The reprints of the papers are attached as well.

4.1 Multichannel blind iterative image restoration

▪ F. Šroubek and J. Flusser, "Multichannel Blind Iterative Image Restoration," *IEEE Trans. Image Processing*, accepted in March 2003. ▪

An iterative algorithm for multichannel blind deconvolution with enhanced noise robustness is presented in this work.

Intrinsically multichannel methods proposed so far [67, 68, 71, 69, 70, 72, 73, 74] are able to recover the original image if at least two images degraded with coprime blurs are available. Their performance however deteriorates rapidly if noise is added to the acquisition model. To overcome this negative behavior that prevents the classical multichannel methods from being used in real applications, we propose a novel constrained minimization problem. The constrained problem is first transformed to an unconstrained minimization problem using the Lagrange multipliers. The resulting energy functional is a function of the estimated original image and of the estimated blurs. It comprises three terms. The first one measures the fidelity of our estimates to the degraded images according to the acquisition model. The second one is an anisotropic regularization term (total variation or Mumford-Shah functional) that prevents undesirable attenuation of edges and fine details in the image. The last term constrains the estimated blurs so that they are consistent with the mutual relation of any two channels, which states that $z_i * h_j - z_j * h_i = 0$ for every $i \neq j$ if noise is zero.

Before we can proceed to a numerical solution, a relaxed version of the energy functional is first formulated. Then a suitable numerical scheme for this type of nonlinear minimization problems is the half-quadratic algorithm, which introduces an auxiliary variable, also called a dual variable. The energy becomes a function of three variables, the original image estimate, blurs estimate and dual variable, but it is convex with respect to each one. Finally, a multichannel alternating minimization (MC-AM) scheme is proposed that performs an analogy of the steepest descent in the space of the image and blurs.

Convergence properties of the described algorithm together with the estimation of two parameters associated with the Lagrange multipliers are discussed in detail.

A large section of the paper is dedicated to experimental results. First, the performance of the MC-AM algorithm is compared with the performance of the subspace method EVAM [67, 68], which belongs to the group of intrinsically multichannel methods. The performance was tested on simulated data for different SNR's and the results clearly favour the MC-AM method, especially for lower SNR's (higher noise levels). The rest of the experiments assess the applicability of MC-AM on real data: out-of-focus images of a flat scene and astronomical images of a sunspot. Apart from the visual assessment of the results, a wavelet-based focus measure was used as an objective measure of the restoration performance. In the case of the out-of-focus images, a remarkably well focused image was obtained from three blurred images that were only slightly different from each other. The same sequence of astronomical images considered here were analyzed in our earlier work; see Section 4.3. However, the sunspot image restored by MC-AM demonstrates the clear superiority of the novel method over the EVAM approach utilized in the earlier work. In all the experiments, we estimated the blur size beforehand.

The proposed iterative algorithm is sufficiently robust to noise and was successfully applied to images acquired in real situations. Nevertheless, an accurate estimation of the blur size is still necessary.

4.2 Multichannel blind deconvolution of spatially misaligned images

■ F. Šroubek and J. Flusser, "Multichannel blind deconvolution of spatially misaligned images," *IEEE Trans. Image Processing*, submitted in March 2003. ■

Intrinsically multichannel blind deconvolution techniques proposed in the literature crucially depend on the correct size of the blurs. To our knowledge, all the techniques also assume that the images are correctly registered. If this is not the case, a registration step must precede restoration. In this paper, we present a multichannel approach that is less dependent on the blur size and that can deal with misregistration of images.

A convolution operator performs translation if the origin of the convolution kernel is shifted. Therefore, if the blurs are sufficiently enlarged so that they include the original blurs properly shifted, deconvolution together with registration is achieved at once. The mutual relation between channels (see Section 4.1) plays the fundamental role in the intrinsically multichannel methods, where it is used for recovering blurs from the degraded images. A proof that the mutual relation can also be used for recovering blurs from spatially misaligned images is given here.

The restoration problem is phrased as a search for a maximum a posterior (MAP) estimate. This stochastic approach refers to discrete images as random vector fields characterized by probability distribution functions. To be able to formulate the MAP estimator, a priori distributions of the original image and blurs are required.

The a priori distribution of the original image is difficult to obtain. A common approach is to model such distributions as Markov Random Fields or Gibbs distributions. A space of bounded variation (BV) functions is considered by many as an acceptable domain for real images, because discontinuities (edges) in images are allowed therein. The notion of total variation, which was introduced with the BV space, will serve the purpose of an energy function in the Gibbs distribution.

The a priori distributions of the blurs is derived directly from the multichannel character of the problem, more precisely, from the mutual relation between channels. Simplifications are however necessary to make this issue more tractable.

After the derivation of both prior distributions, an alternating minimization scheme (AM-MAP) is proposed to solve the MAP estimation.

A large section of the paper is devoted to a convergence analysis and to experiments on simulated and real data. The convergence and average performance of the AM-MAP algorithm for different SNR's and for different degrees of the blur size overestimation are presented. A comparison of AM-MAP with the most recent intrinsically multichannel approach of Pai [73, 74] was conducted as well. The performance was tested on simulated data for different SNR's. The restoration error of AM-MAP is smaller then the error of Pai's method on the entire interval of tested SNR's. Finally to demonstrate the applicability of the AM-MAP approach on real misaligned data, an experiment with images degrade by random vibration blurs was performed.

We may conclude that the proposed MAP method gives us a sound solution to the restoration problem of images, which are not only degraded by blurs and noise but also misregistered.

4.3 Multichannel blind deconvolution of the short-exposure astronomical images

F. Šroubek, J. Flusser, T. Suk, and S. Šimberová, “Multichannel Blind Deconvolution of the short-exposure astronomical images,” in *Proc. of the 15th International Conference on Pattern Recognition*, pp. 53–56, 2000.

This conference paper illustrates one of the first attempts to apply an intrinsically multichannel method to restoration of astronomical images. Real astronomical data under investigation were obtained from an observation of the Sun with a terrestrial telescope. The acquired short-exposure images of a sunspot in the visible spectral band were taken one after another in short time intervals and were degraded not only by the intrinsic PSF of the telescope but primarily by random perturbations of wavefronts in the Earth atmosphere. In the visible light, the effects of fluctuations in the refractive index of the air caused by temperature variations are profound. Since the atmospheric conditions may change very quickly, the acquired image sequence usually contains images of different quality from almost sharp to heavy blurred ones. This type of data are well registered and satisfy the fundamental assumption of intrinsically multichannel methods, which states that the channel PSF’s are “sufficiently diverse” (coprime). Moreover, the least degraded images can be regarded as reference images and used for assessment purposes.

The algorithm proposed here is based on an intrinsically multichannel approach of Harikumar and Bresler [67, 68] called the EVAM subspace method. The EVAM method is used to estimate, for a given blur size, the PSF’s from the blurred images. Since the estimate very much depends on the blur size, a full search is performed over an admissible set of blur sizes. For each set of PSF’s multichannel nonblind deconvolution is calculated and by means of a standard residual function the least erroneous restored image is selected. As an objective measure of the restoration performance an integral of a sum of image partial derivatives is considered. Visual assessment is conducted as well.

This work belongs to the earlier ones. Although we have not suggested a new multichannel method, the contribution is that the applicability of the previously proposed multichannel approach was verified on astronomical data. The results stated here are fully compliant with the thesis’ objective of demonstrating the capability of the multichannel restoration methods on data obtained from real applications.

4.4 Application of image processing to the medieval mosaic conservation

- B. Zitová, J. Flusser, and F. Šroubek, “Application of Image Processing to The Medieval Mosaic Conservation,” *Pattern Analysis and Applications*, submitted.

In this work, we demonstrate how up-to-date image enhancement and registration techniques can be utilized to validate such delicate conservation tasks as the reconstruction of the 14^{th} century mosaic is. Uniqueness of the described application lies in a fact that this splendid marvel made by medieval artists was captured on an old photograph just before it was severely damaged and removed from the wall, where is was situated for several centuries, in the second half of the 19^{th} century. This little coincident contains precious information that enables us to compare the current renovated mosaic with its formal state and to evaluate any discrepancies introduced by several conservation efforts in the last 120 years. The problem we face here consists of three steps: enhancement of the old photograph, geometric alignment of the old photograph with the new one depicting the mosaic after the last renovation and identification of any differences between the two photographs.

The subject of the enhancement task is the old photograph degraded primarily due to aging effects (diffusion of chemical compounds in old photographic material). This problem is singlechannel in its nature and the intrinsically multichannel methods can not be applied therefore. We first considered advanced anisotropic denoising techniques, like wavelet-based and adaptive nonlinear filters, and satisfying results were obtained. Afterwards we applied several singlechannel deconvolution methods in both the blind and nonblind setting. For the nonblind case, Gaussian blurs of different variance were tested.

Feature based restoration was considered for the registration task. Since the color tones of the old photograph do not match the color tones of the new one, correlation methods for feature matching can not be applied. Instead, a manual selection and correspondence of salient points was used. A method based on mutual information performed additional refinement of the point locations and a geometric transformation between the photographs was then calculated.

Using modern image processing methods for denoising, deconvolution and image registration, we were able to successfully identify pattern differences overlooked by restorers. We claim that digital image processing can provide important complementary data for scholars and should be thus considered as a verification tool for such restoration tasks.

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