Interface tightness

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Tuesday, March 26th, 2019

Something else for a start

We find good friends among the artists and progressive people of diverse continents, to whom we are tied by the ideas of true humanism, the possibility to ensure work, freedom and security in life, and to satisfy the material and spiritual needs not of a chosen handful, but of millions of people.
That is why –in line with the Final Act of the Helsinki Conference– we stretch out our hand across the borders of countries and continents, fully aware that true art and true culture should help individual nations and all of humanity move forward; they should create understanding among people of diverse countries; they should win people over to the humanistic perspective on peace and mutual cooperation in the interest of happy human life. . . .

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- Informally known as the *Anticharter*.
- Why *anti?* It seems all so positive. All, except for *a few lines...*

Something else for a start

. . . That is why we despise those who, in unbridled pride, in a vain feeling of superiority, for selfish interests, or even for filthy financial gain in various places all over the world –and even in our land a small group of such dropouts and traitors can be found– divorce and isolate themselves from their own people and its life and real interests and, with inexorable logic, become instruments of the antihumanistic forces of imperialism and, in its service, the heralds of disruption and discord among nations.

Something else for a start

Charter 77

242 signatories, including the later president Václav Havel:

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- Claim tens of thousands cannot work in their own field because they support opinions different from the official opinions.
- Claim hundreds of thousands live in fear that for venting their opinions, they can be excluded from the work or study of their choice.

Something else for a start

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- Some of those who tried to further publicize individual cases of repression were sentenced to prison sentences of a few years.
- Artists, who refused to sign the “anticharter”, where excluded from their jobs.
- It was forbidden to spread the Charter 77. Even artists who signed the “anticharter” could not read Charter 77.

Something else for a start

We write here first to state, in the strongest possible terms, that the humanity of any person, regardless of ascribed identities such as race, ethnicity, gender identity, religion, disability, gender presentation, or sexual identity is not up for debate. Physics and science are part of the shared inheritance of all people, as much as art, music, and literature, and we should strive to ensure that everyone has a fair opportunity to become a scientist. The question of discrimination based on ascribed identity is a moral one, and we write to affirm that discrimination is not a welcome feature of our field, however pervasive it may be. . . .

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- Aimed at what?

Something else for a start

. . . Secondly, we write to strongly express our view that the science case presented by Strumia was fundamentally unsound. It is clear to all of us that Strumia is not an expert on these topics and is misusing his physics credentials to put himself forward as one. Furthermore, those among us who are familiar with the relevant literature know that Strumia's conclusions are in stark disagreement with those of experts. He frequently made the basic error of conflating correlation with causation, and while Strumia claimed to be proving that there is no discrimination against women, his arguments were rooted in a circumscribed, biased reading of the data available, to the point of promoting a perspective that is biased against women.

Something else for a start

All against *one* man: **Alessandro Strumia** (Pisa).

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- **1.** There is no significant discrimination against women in theoretical physics.

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- **2.** Women are underrepresented in theoretical physics because there are fewer of them that are good at it.

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- **1.** There is no significant discrimination against women in theoretical physics.
- **2.** Women are underrepresented in theoretical physics because there are fewer of them that are good at it.
- **3.** There is discrimination against men by hiring committees in theoretical physics.

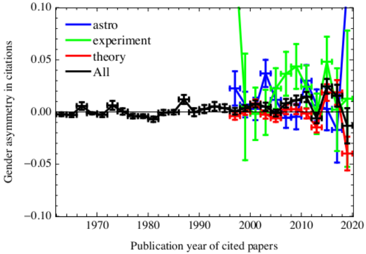
Something else for a start

Count single-author that cites different single-author, define

$$\text{Gender asymmetry} = A \equiv \frac{1}{N^{\text{cit}}_{M\to}N^{\text{cit}}_{F\to}} \det\begin{pmatrix} N^{\text{cit}}_{M\to M} & N^{\text{cit}}_{M\to F} \\ N^{\text{cit}}_{F\to M} & N^{\text{cit}}_{F\to F} \end{pmatrix} \qquad -1 \le A \le 1$$

InSpire, after 2010

category	A
hep-ex	$(-1.2 \pm 1.7)\%$
hep-ph	$(0.1 \pm 0.6)\%$
hep-th	$(-0.1 \pm 0.7)\%$,
astro-ph	$(0.6 \pm 1.1)\%$
hep-lat	$(0.2 \pm 2.2)\%$
nucl-ex	$(0.2 \pm 2.1)\%$
gr-qc	$(0.2 \pm 1.2)\%$



No gender preference in citations in any category at any time, down to % level.

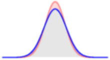
Similar analysis applied to countries finds instead significant asymmetries.

Arguments for point 1.

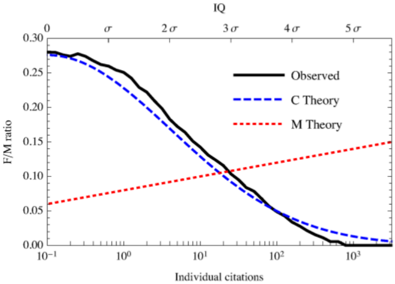
Something else for a start

Physics graduates have top ⟨IQ⟩, it's needed. Men have similar ⟨IQ⟩ as women and ∼ 15% higher standard deviation ('diversity'), as in other traits. C predicts:

$$\frac{N_F}{N_M} = \text{(interest)} \times \text{(ability)} \approx \frac{1}{4} \times \frac{\text{Gaussian}(\sigma = 1)}{\text{Gaussian}(\sigma = 1.15)} = \frac{1}{4} \times$$



Convert ⟨IQ⟩ to $\langle N_{\text{icit}} \rangle$ assuming one 6σ among 10^9 persons:



Arguments for point 2.

Something else for a start

INFN positions in theory, 2018. Gender 'experts' only:

Role	Name	N_{cit}
Commissar	Silvia Penati	2130
Hired	Anna Ceresole	3231
Not Hired	Alessandro Strumia	30785

And finally arguments for point 3, based on a sample of $n = 1$.

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All I want to say: Sometimes the *methods* we use to achieve an aim are more important than *the aim itself.*

One-dimensional voter models

$\{0,1\}^{\mathbb{Z}}$ = the space of all functions $x:\mathbb{Z}\to\{0,1\}$. Interpretation:

$$x = \cdots 00001101010001101011100111111\cdots$$

models the distribution of two genetic types of a plant, living in a one-dimensional environment (coastline, river).

$(X_t)_{t\geq 0}$ with $X_t = \big(X_t(i)\big)_{i\in\mathbb{Z}}$ continuous-time Markov process with state space $\{0,1\}^{\mathbb{Z}}$.
Dynamics: each plant lives an exponential time with mean 1, and upon death is immediately replaced by a clone of a near-by plant, at a distance chosen according to a probability distribution p.

In other words, if the present state is x, then $x(i)$ jumps:

$$\begin{aligned} 0 &\mapsto 1 \qquad \text{with rate} \qquad \sum_{j\in\mathbb{Z}} p(j-i)1_{\{x(j)=1\}}, \\ 1 &\mapsto 0 \qquad \text{with rate} \qquad \sum_{j\in\mathbb{Z}} p(j-i)1_{\{x(j)=0\}}. \end{aligned}$$

Interfaces

$$\begin{aligned} S^{01}_{\rm int} := \big\{x \in \{0,1\}^{\mathbb{Z}} : \exists i < j \text{ s.t. } & x(i') = 0 \ \forall i' \leq i, \\ & x(j') = 1 \ \forall j' \geq j\big\}. \end{aligned}$$

Interpretation: $x \in S^{01}_{\rm int}$ describes the *interface* between two infinite populations of 0's and 1's:

$$\cdots 0000000000000 \underbrace{1011000110100}_{\text{interface}} 11111111111111 \cdots$$

Lemma

If $\displaystyle\sum_k p(k)|k| < \infty$, then $X_0 \in S^{01}_{\rm int}$ implies $X_t \in S^{01}_{\rm int}$ $\forall t \geq 0$ a.s.

Question Starting from the *Heaviside configuration*

$$x_0 := \cdots 0000000000000000000000111111111111111111111 \cdots$$

does the size of the interface keep growing, or does it reach some finite equilibrium size?

Numerics

A voter model on $\{1, \ldots, 500\}$ with periodic boundary conditions, and p the uniform distribution on $\{-2, -1, 1, 2\}$. Total time elapsed 600.

Interface tightness

Def $x \sim y$ if $\exists j$ s.t. $x(i) = y(i+j)$ $(i \in \mathbb{Z})$.

Def $\overline{x} := \{y : y \sim x\}$ and $\overline{S}^{01}_{\rm int} := \{\overline{x} : x \in S^{01}_{\rm int}\}$.

Observation The *voter model modulo translations* $(\overline{X}_t)_{t \geq 0}$ is a Markov process.

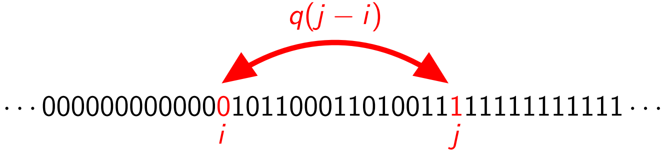
Def A voter model exhibits *interface tightness on* $S^{01}_{\rm int}$ if $\overline{x}_0$ is a positive recurrent state for the Markov process $(\overline{X}_t)_{t \geq 0}$.

Theorem If $\displaystyle\sum_k p(k)|k|^2 < \infty$, then interface tightness holds on $S^{01}_{\rm int}$ and $S^{10}_{\rm int}$.

Proved when $\sum_k p(k)|k|^3 < \infty$ by Cox and Durrett (1995) and in general by Belhaouari, Mountford and Valle (2007), who moreover showed that the second moment condition is optimal.

Voter-exclusion models

In an *exclusion process*, the states of two sites i and j are interchanged with rate $q(j-i)$, where $q : \mathbb{Z} \to [0, \infty)$ is a symmetric function.



$q(j-i)$

$\cdots 000000000000\underset{i}{0}101100011010011\underset{j}{1}11111111111 \cdots$

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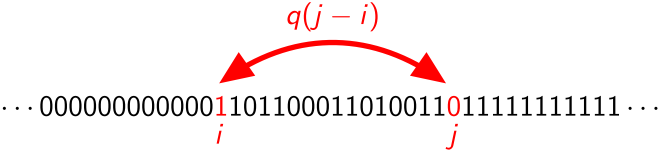

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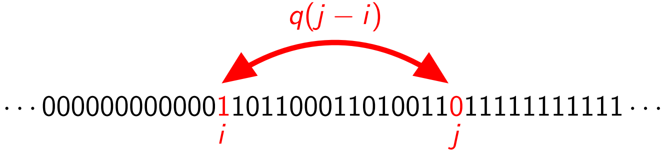


Theorem [Sturm & S. '08] If $\sum_k p(k)|k|^2<\infty$ and $\sum_k q(k)|k|^2<\infty$, then interface tightness on $S^{01}_{\rm int}$ and $S^{10}_{\rm int}$ holds for voter-exclusion processes.

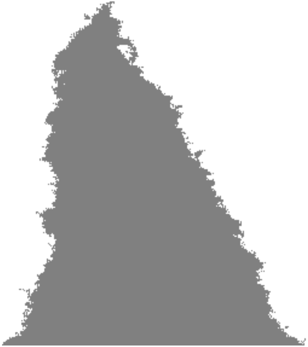
Biased voter models

In the *biased voter model* with *bias* $\varepsilon \in [0,1]$, $x(i)$ jumps:

$$\begin{array}{lll} 0 \mapsto 1 & \text{with rate} & \displaystyle\sum_{j\in\mathbb{Z}} p(j-i)1_{\{x(j)=1\}}, \\ 1 \mapsto 0 & \text{with rate} & \displaystyle(1-\varepsilon)\sum_{j\in\mathbb{Z}} p(j-i)1_{\{x(j)=0\}}. \end{array}$$

Theorem [Sun, S. & Yu '18] If $\displaystyle\sum_{k<0} p(k)|k| < \infty$ and $\displaystyle\sum_{k>0} p(k)|k|^2 < \infty$, then interface tightness holds on $S^{01}_{\rm int}$.

Numerics



A biased voter model with bias $\varepsilon = 0.3$.

Rebellious voter models

Let $f_\sigma(i) := \sum_{j\in\mathbb{Z}} p(j-i)1_{\{x(j)=\sigma\}} \quad (\sigma = 0,1).$

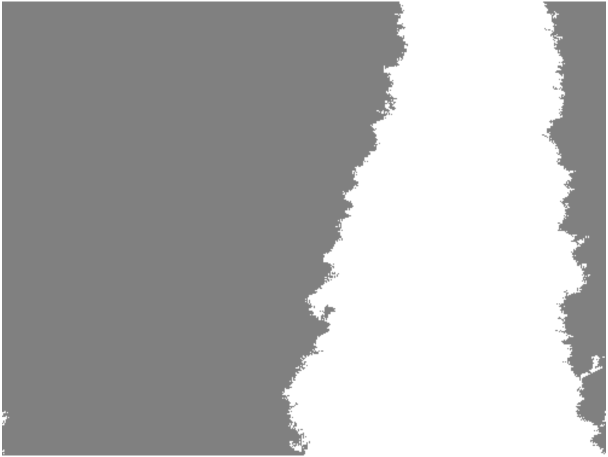
In the neutral *Neuhauser-Pacala model* (1999) with *competition parameter* $\alpha \geq 0$,

$$x(i) \text{ flips } \begin{cases} 0 \mapsto 1 \text{ with rate } f_1(i)\big(f_0(i)+\alpha f_1(i)\big), \\ 1 \mapsto 0 \text{ with rate } f_0(i)\big(f_1(i)+\alpha f_0(i)\big). \end{cases}$$

Note: $f_0(i)+f_1(i)=1$. For $\alpha=1$ this is the voter model.
For $\alpha<1$, you die faster if your neighbors are of *your own* type.
For $\alpha>1$, you die faster if your neighbors are of *the other* type.

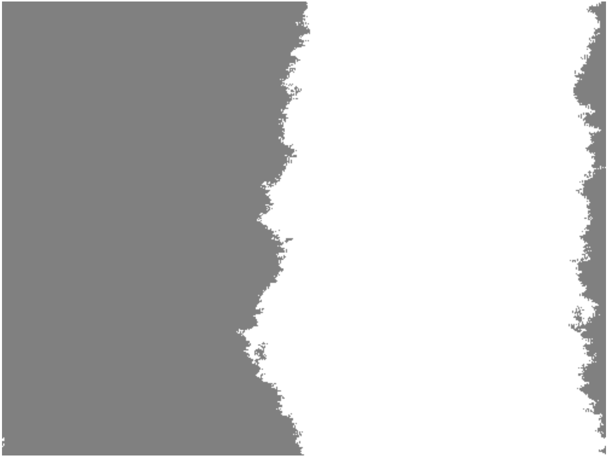
Conjecture There exists an $\alpha_{\rm c}<1$ such that interface tightness holds for $\alpha>\alpha_{\rm c}$, but not for $\alpha\leq\alpha_{\rm c}$.

Numerics



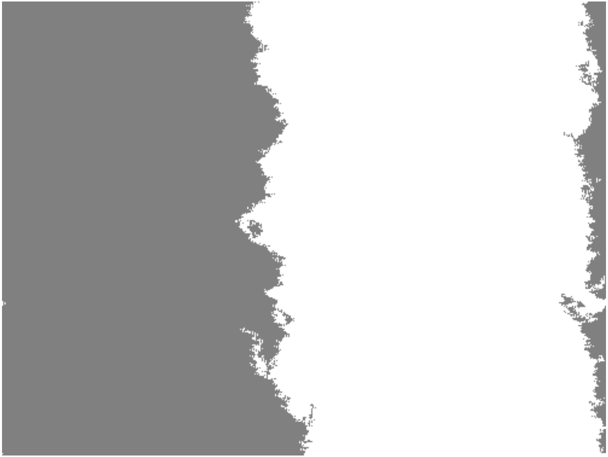
A neutral Neuhauser-Pacala model with $\alpha = 0.9$.

Numerics



A neutral Neuhauser-Pacala model with $\alpha = 0.8$.

Numerics

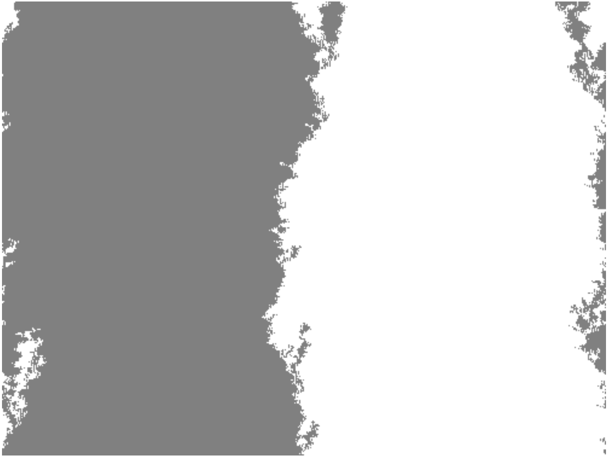


A neutral Neuhauser-Pacala model with $\alpha = 0.7$.

Numerics

A neutral Neuhauser-Pacala model with $\alpha = 0.6$.

Numerics



A neutral Neuhauser-Pacala model with $\alpha = 0.5$.

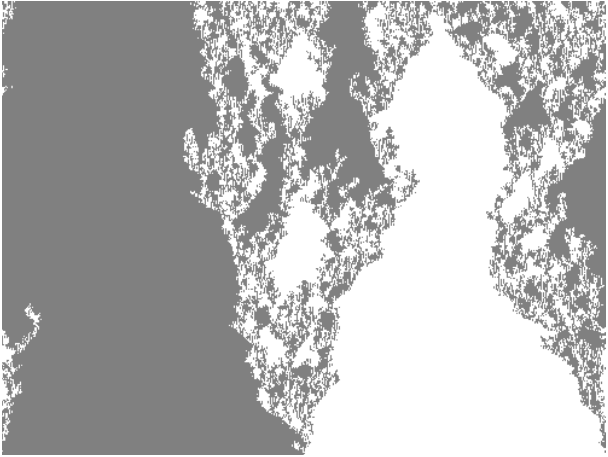
Numerics

A neutral Neuhauser-Pacala model with $\alpha = 0.4$.

Numerics

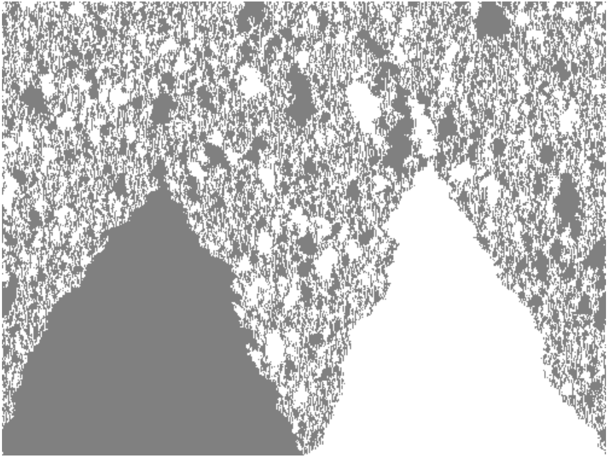
A neutral Neuhauser-Pacala model with $\alpha = 0.3$.

Numerics



A neutral Neuhauser-Pacala model with $\alpha = 0.2$.

Numerics

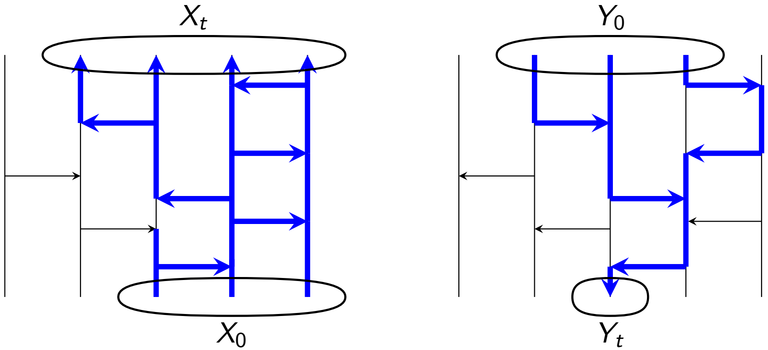


A neutral Neuhauser-Pacala model with $\alpha = 0.1$.

Other models

- Asymmetric exclusion processes. [Bramson, Liggett & Mountford '02].
- Neutral two-type contact processes. [Mountford & Valesin '10, '16].
- Biased two-type contact processes. [Andjel, Mountford, Pimentel & Valesin '10]

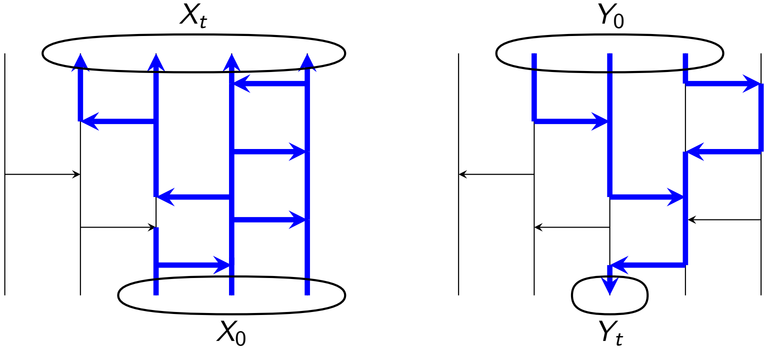
Duality



A voter model X is dual to a system of coalescing random walks Y:

$$\mathbb{P}\big[X_t \wedge Y_0 \neq 0\big] = \mathbb{P}\big[X_0 \wedge Y_t \neq 0\big] \quad (t \geq 0).$$

Duality



Interpretation: Set $\mathcal{Y}_t := \{i : Y_t(i) = 1\}$.
Then $\mathcal{Y}_t$ are the ancestors of $\mathcal{Y}_0$.
$X_t(i) = 1$ for some $i \in \mathcal{Y}_0$ iff $X_0(i) = 1$ for some $i \in \mathcal{Y}_t$.

A useful function

Cox and Durrett (1995) look at the function

$$h(x) := \sum_{i<j} 1_{\{x(i) > x(j)\}} \qquad \big(x \in S^{01}_{\rm int}\big),$$

which counts the *number of inversions.* For the process started in the Heaviside state x_0, they used duality to prove

$$\sup_{t\geq 0} \mathbb{P}\big[h(X_t) \geq N\big] \underset{N\to\infty}{\longrightarrow} 0.$$

The function h also plays a key role in the proofs of Belhaouari, Mountford and Valle (2007).

The width of the interface

Let $\ell(x) := \sup\{i : x(i) = 0\} - \inf\{i : x(i) = 1\} + 1$ denote the *width of the interface.*

$$\cdots 0000000000000\,\underbrace{10110}_{\ell(x)=5}\,11111111111111\cdots$$

If interface tightness holds, then $\overline{X}_t$, started in $\overline{x}_0$, converges in law as $t \to \infty$ to some $\overline{X}_\infty$. Cox and Durrett (Theorem 6) prove that

$$\mathbb{E}\big[\ell(\overline{X}_\infty)\big] = \infty.$$

Belhaouari, Mountford, Sun and Valle (2006, Theorem 1.4) have shown that

$$\mathbb{E}\big[\ell(\overline{X}_\infty) \geq L\big] \asymp L^{-1}.$$

A pseudo-Lyapunov function

The process modulo translations $\overline{X}_t$ is a countinuous-time Markov chain with countable state space $\overline{S}^{01}_{\rm int}$.

By Foster's theorem, positive recurrence is equivalent to the existence of a Lyapunov function $V : \overline{S}^{01}_{\rm int} \to [0, \infty)$ such that

$$\begin{aligned} GV(x) &< \infty \qquad && \text{for all } x \in \overline{S}^{01}_{\rm int}, \\ GV(x) &\leq -1 \qquad && \text{for all but finitely many } x \in \overline{S}^{01}_{\rm int}, \end{aligned}$$

where G is the *generator* of $\overline{X}_t$.

For the voter model modulo translations, no such Lyapunov function has been found explicitly.

A pseudo-Lyapunov function

Sturm & S. (2008) have shown that the *number of inversions* $h(x)$ is "almost" a Lyapunov function.

More precisely,

$$Gh(x) = \tfrac{1}{2}\sum_{k\in\mathbb{Z}} p(k)|k|^2 - \tfrac{1}{2}\sum_{k\in\mathbb{Z}} p(k)I_k(x),$$

where

$$I_k(x) := \sum_{i\in\mathbb{Z}} 1_{\{x(i)\neq x(i+k)\}}$$

denotes the number of *k-boundaries.*

Since $\{\overline{x} : x \in S^{01}_{\rm int},\ Gh(x) \not\leq -1\}$ is in general not finite (except when p is almost nearest neighbor),
this is not a Lyapunov function.

A pseudo-Lyapunov function

Nevertheless, it is almost as good as a Lyapunov function. One can show that if interface tightness does not hold, then

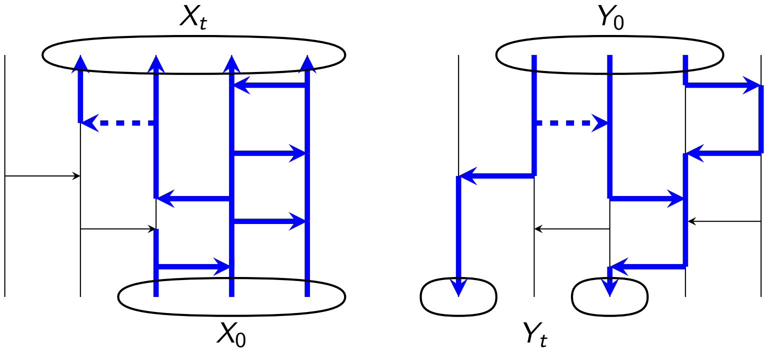
$$\lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\ \mathbb{P}\big[I_k(X_t)<N\big]=0 \qquad (N,k\geq 1),$$

i.e., *most of the time, there are lots of k-boundaries.*

As a result, most of the time $Gh(X_t)\leq -1$, while the rest of the time $Gh(X_t)\leq\frac{1}{2}\sum_{k\in\mathbb{Z}}p(k)|k|^2<\infty$.

This means that if interface tightness does not hold, then over long time intervals, $h(X_t)$ decreases more than it increases. Since $h\geq 0$, we arrive at a contradiction. ■

Duality for biased voter models



A biased voter model X has a branching-coalescing dual Y:

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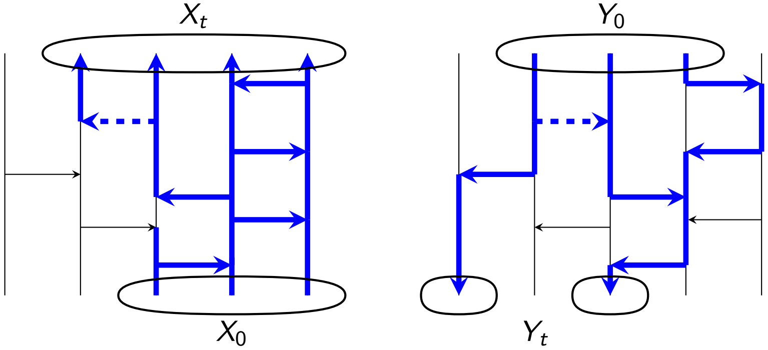

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Interface tightness for biased voter models

Sun, S. & Yu (2018) prove interface tightness for biased voter models using the pseudo-Lyapunov function technique of Sturm & S. Set:

$$i_0(x) := \inf\{i \in \mathbb{Z} : x(i) = 1\},$$
$$i_{n+1} := \inf\{i > i_n : x(i) = 1\}.$$

A suitable pseudo-Lyapunov function turns out to be the *weighted number of inversions*

$$h_\varepsilon(x) := \sum_{n=0}^{\infty} (1-\varepsilon)^n \sum_{j > i_n} 1_{\{x(j) = 0\}}.$$

Scaling limit

Let L_t and R_t denote the left and right boundaries

$$L_t := \inf\{i \in \mathbb{Z} : X_t(i) = 1\} \quad \text{and} \quad R_t := \sup\{i \in \mathbb{Z} : X_t(i) = 0\}.$$

For the unbiased voter model, it has been proved by Belhaouari, Mountford, Sun and Valle (2006) that

$$\big(\varepsilon L_{\varepsilon^{-2}t}, \varepsilon R_{\varepsilon^{-2}t}\big) \underset{\varepsilon\to 0}{\Longrightarrow} \big(B_t, B_t\big)_{t\geq 0},$$

where B_t is a Brownian motion with variance $\sum_k p(k)|k|^2$.

For the biased voter model with bias ε, a similar result has been conjectured where B_t is a drifted Brownian motion.

Heuristics

Recall that the equilibrium interface width is heavy-tailed:

$$\mathbb{E}\big[\ell(\overline{X}_\infty) \geq L\big] \asymp L^{-1}.$$

On the other hand, numerics suggest that the equilibrium number of 1-boundaries has an exponential tail:

$$\mathbb{E}\big[I_1(\overline{X}_\infty) \geq N\big] \asymp e^{-cN} \quad \text{for some } c > 0.$$

Conjecture:

$$\lim_{L\to\infty} \mathbb{P}\big[I_1(\overline{X}_\infty) \in \cdot \,\big|\, \ell(\overline{X}_\infty) \geq L\big]$$

exists, and equals the law of $I^1 + I^2 + I^3$, where I^1, I^2, I^3 are i.i.d. and equally distributed with $I_1(\overline{X}_\infty)$.

Proof?

Three Brownian motions

Let $(B^1_t, B^2_t, B^3_t)_{t\geq 0}$ be three independent Brownian motions started from $B^i_0 = x_i$, with $x_1 < x_2 < x_3$. Set

$$\tau := \tau_{12} \wedge \tau_{23} \quad \text{with} \quad \tau_{ij} := \inf\{t \geq 0 : B^i_t = B^j_t\}.$$

Then it is known (Grabiner 1999) that

$$\mathbb{P}[\tau > t] \sim \frac{3}{4\sqrt{\pi}} \prod_{1\leq i<j\leq 3} (x_j - x_i)\, t^{-3/2} \qquad \text{as } t \to \infty.$$

In particular, τ has finite mean.

Heuristics

We want to heuristically "deduce" that

$$\mathbb{E}\big[\ell(\overline{X}_\infty) \geq L\big] \asymp L^{-1}.$$

By definition, a macroscopic "excursion" of duration τ is an event where a microscopic interface splits into three interfaces. After time τ, two of these interfaces annihilate each other, leaving only one microscopic interface.
For a given excursion, $\mathbb{P}[\tau > T] \sim T^{-3/2}$.
In equilibrium, the duration $\hat{\tau}$ of the excursion that takes place at time zero has a size-biased law, so $\mathbb{P}[\hat{\tau} > T] \sim T^{-1/2}$.
The width ℓ of an excursion of duration $\hat{\tau}$ should be of order $\ell = \sqrt{\hat{\tau}}$, so

$$\mathbb{P}[\ell > L] \approx \mathbb{P}[\sqrt{\hat{\tau}} > L] = \mathbb{P}[\hat{\tau} > L^2] \sim L^{-1}.$$