The Stigler-Luckock model for the evolution of an order book

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Toronto, July 14th, 2016.

Some classical ecomomic theory

In classical economic theory (Walras,[1] 1874), the *price* of a commodity is determined by *demand* and *supply*.

Let $\lambda_-(x)$ (resp. $\lambda_+(x)$) be the total *demand* (resp. *supply*) for a commodity at price level p, i.e., the total amount that people are willing to buy (resp. sell), per unit of time, for a price of at most (resp. at least) p per unit.

Assume that λ_- and λ_+ are continuous, strictly de/increasing, and zero at the endpoints of the price interval I.

Postulate In an equilibrium market, the commodity is traded at the *equilibrium prize* $x_{\rm W}$ defined by $\lambda_-(x_{\rm W}) = \lambda_+(x_{\rm W})$ and the total volume of trade is given by $V_{\rm W} := \sup_{x\in I} \lambda_-(x) \wedge \lambda_+(x)$.

[1] Walras developed the theory of equilibrium markets in his book *Éléments d' économie politique pure.*

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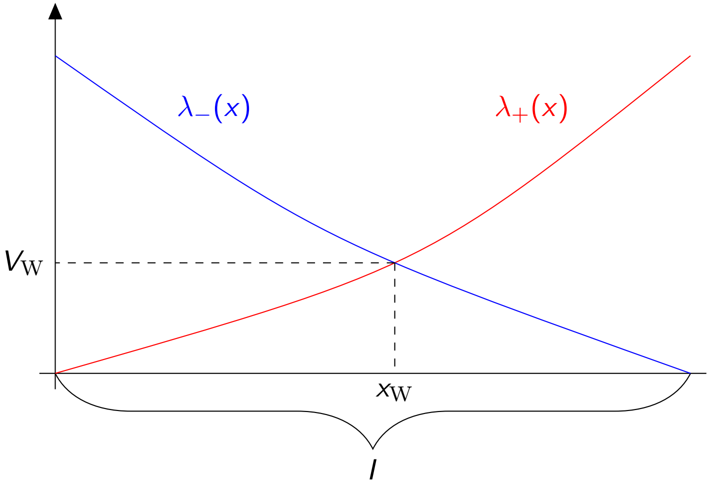

Stock & Commodity Exchanges & the Order Book

On stock & commodity exchanges, goods are traded using an *order book*.

The order book for a given asset contains a list of offers to buy or sell a given amount for a given price. Traders arriving at the market have two options.

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Market orders are matched to existing limit orders according to a mechanism that depends on the trading system.

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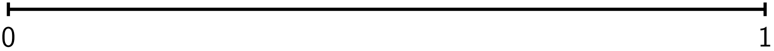
- If the order book contains a suitable offer, then the trader places a *market order*, i.e., buys from the cheapest seller or sells to the highest bidder.
- If the order book contains no suitable offer, then the trader places a *limit order* at his/her maximal buy or minimal sell price.

The uniform model

In the *uniform model*, the price interval is $I = [0,1]$, $\lambda_-(x) = 1 - x$, and $\lambda_+(x) = x$. For this model, $\mu_- = \mu_+$ is the Lebesgue measure on $[0,1]$, and $x_{\rm W} = 0.5$, $V_{\rm W} = 0.5$.

Every model for which $\mu_\pm$ are atomless and $\mu_- = \mu_+$ can be transformed into the uniform model. In general, we can transform any model into a model for which $\mu_- + \mu_+$ is the Lebesgue measure.

Numerical simulation



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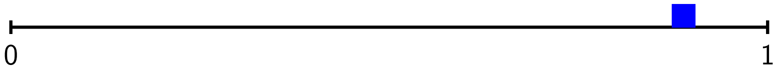

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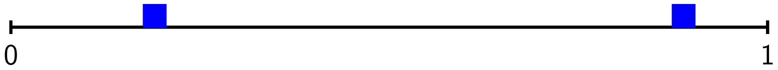

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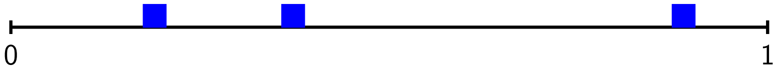

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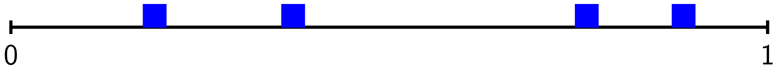

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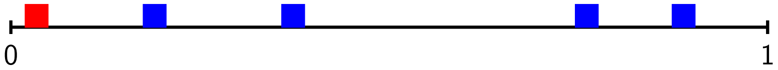

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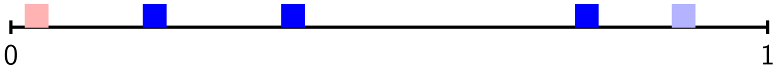

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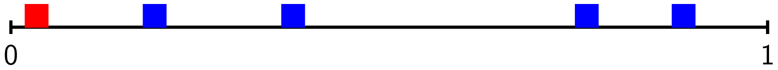

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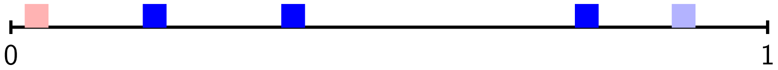

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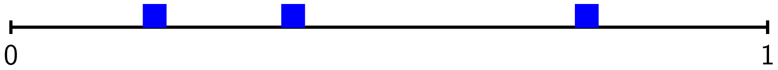

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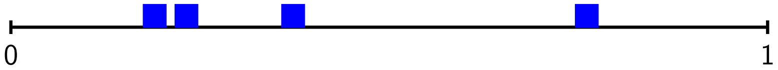

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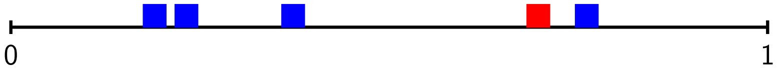

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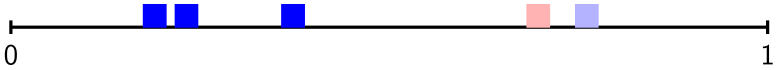

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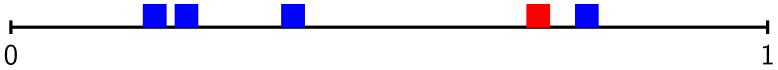

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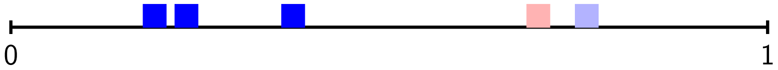

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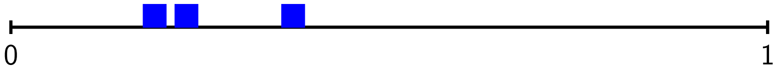

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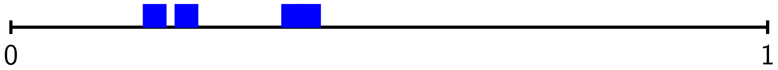

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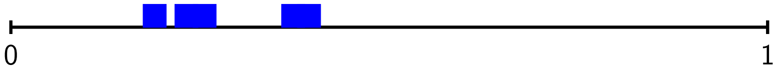

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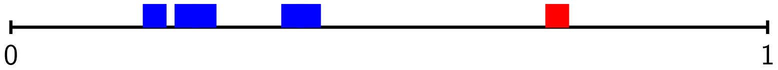

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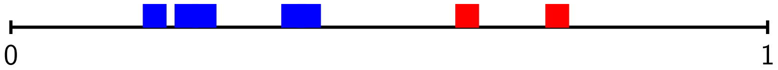

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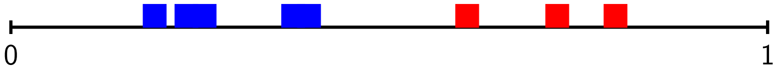

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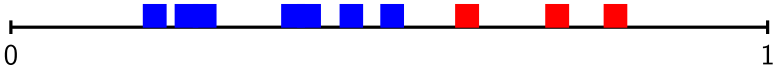

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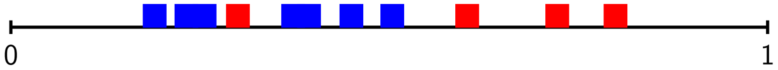

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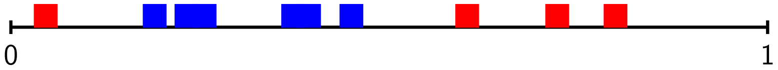

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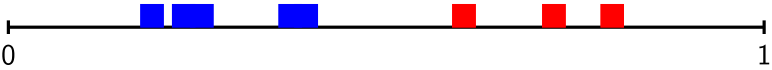

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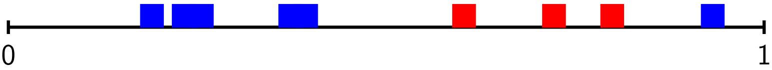

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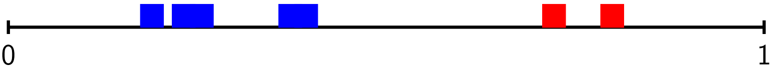

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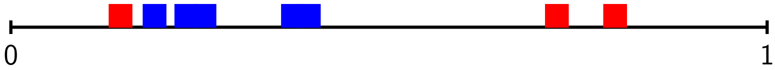

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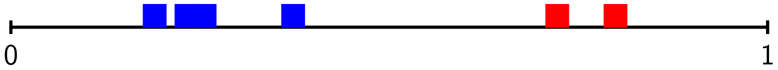

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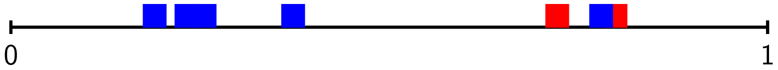

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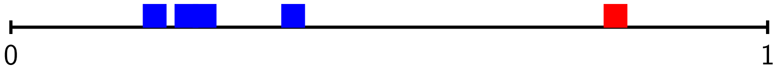

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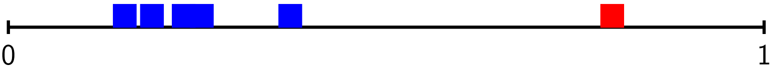

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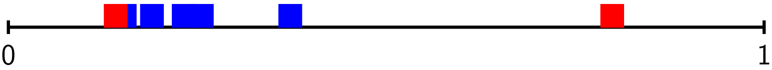

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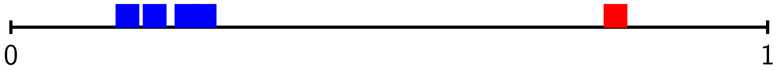

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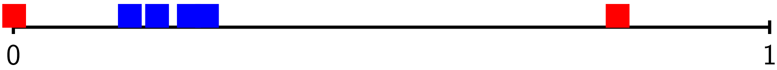

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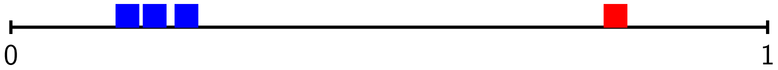

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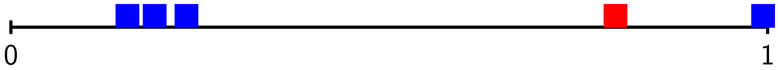

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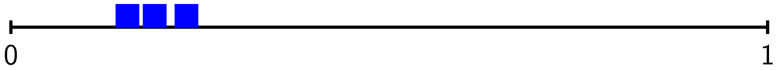

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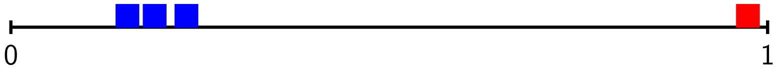

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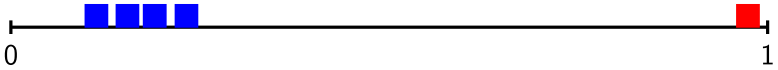

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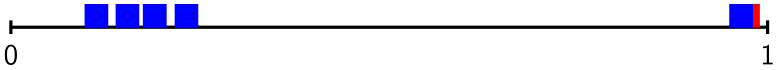

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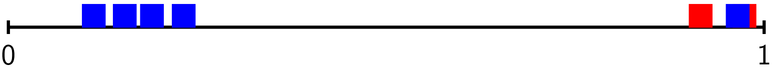

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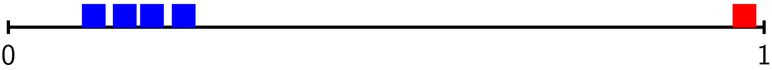

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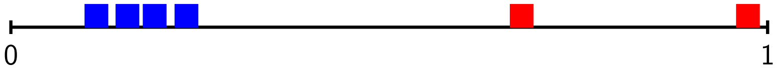

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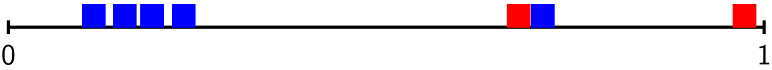

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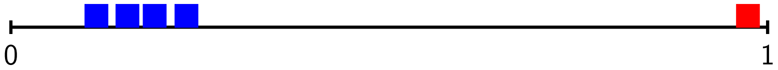

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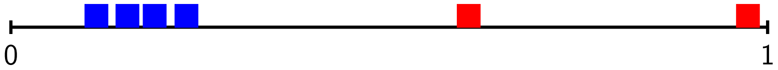

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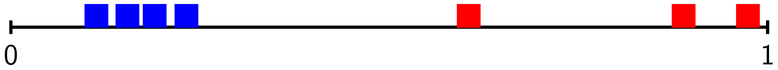

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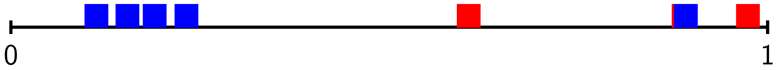

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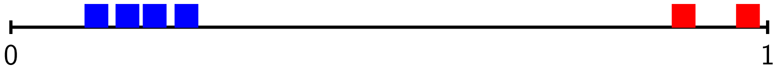

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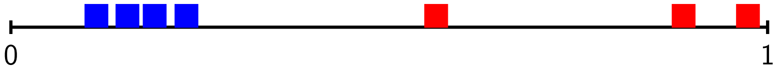

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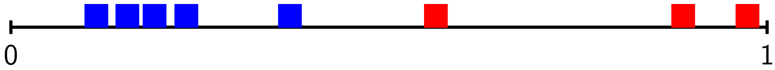

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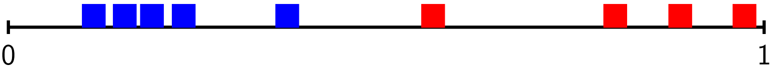

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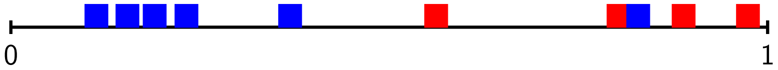

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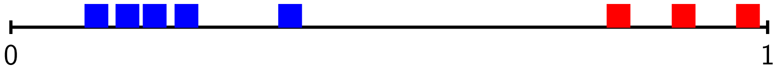

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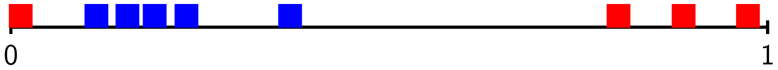

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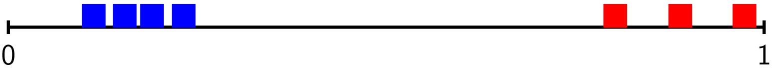

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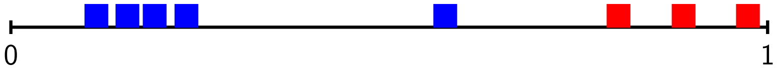

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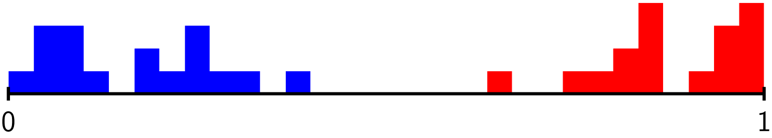
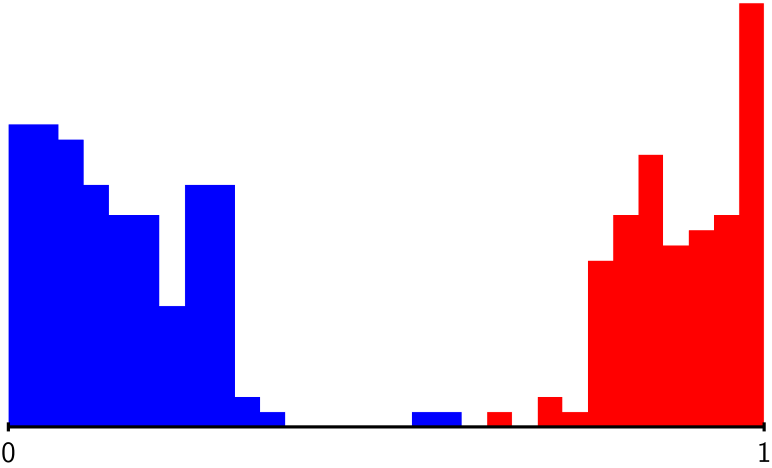


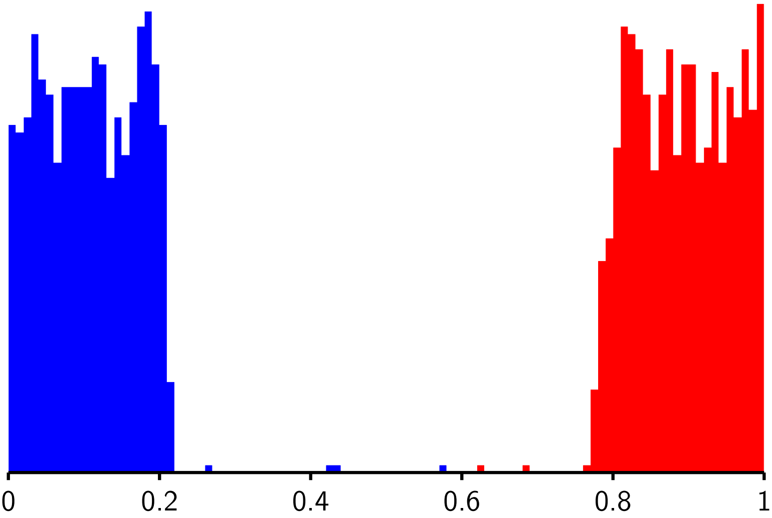
The order book after the arrival of 100 traders.

Numerical simulation



The order book after the arrival of 1000 traders.

Numerical simulation



The order book after the arrival of 10,000 traders.

Stigler's model

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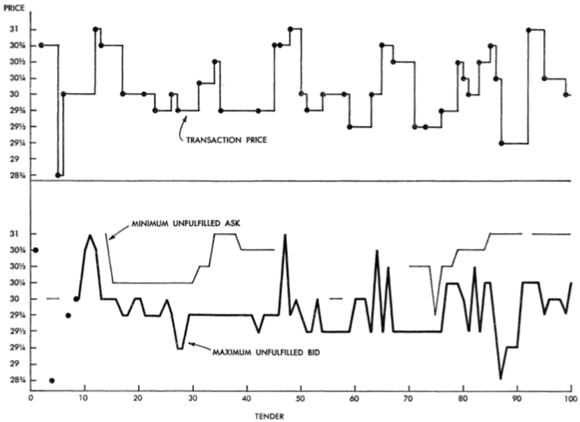
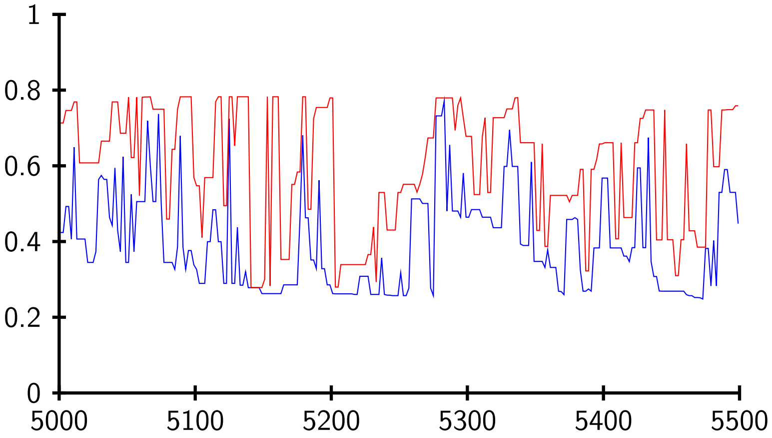


FIG. 1.—Hypothetical sequence of transaction prices, generated by sequence of random numbers, and maximum unfulfilled bid and minimum unfulfilled ask prices (equilibrium price of 29¾ or 30).

Stigler (1964) simulated the model with $\mu_\pm$ the uniform distributions on a set of 10 possible prices.

Numerical simulation



Evolution of the highest bid and lowest ask prices between the arrivals of the 5000th and 5500th trader.

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- The spread is huge, most of the time.

Restricted models

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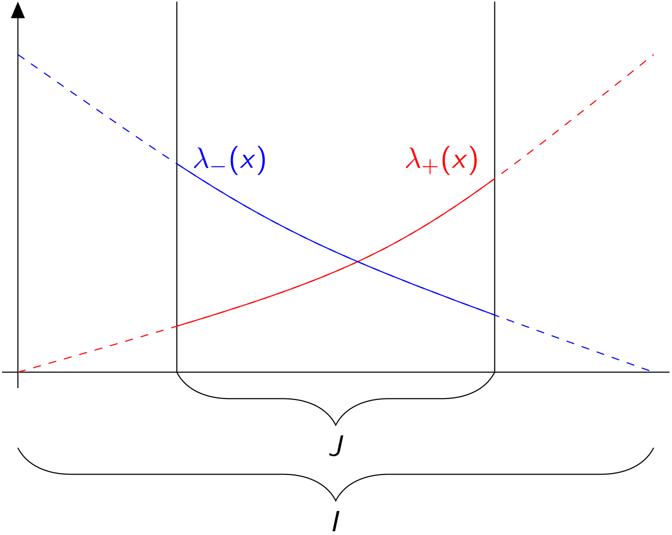
- As long as the bid and ask prices stay inside J, the restricted model behaves the same as the original model.
- For the restricted model, it is no longer true that $\lambda_\pm$ are zero at the endpoints of the interval.

Restricted models



Restricted models

We model the state of the order book by a signed counting measure of the form

$$\mathcal{X} = \sum_i \sigma_i \delta_{x_i} \quad \text{with} \quad x_i \in J,\ \sigma_i \in \{-1, +1\}.$$

We interpret $-\delta_x$ as a buy and $+\delta_x$ as a sell limit order at the price x and write $\mathcal{X} = \mathcal{X}^+ - \mathcal{X}^-$ with $\mathcal{X}^\pm$ disjoint nonnegative measures.

The state space $\mathcal{S}_{\rm ord}$ consists of all $\mathcal{X}$ such that

(i) there are no $x, y \in J$ such that $x < y$, $\mathcal{X}(\{x\}) > 0$, $\mathcal{X}(\{y\}) < 0$.

(ii) $\mathcal{X}^-\big([J_- + \varepsilon, J_+)\big) < \infty$ for all $\varepsilon > 0$.

(iii) $\mathcal{X}^+\big((J_-, J_+ - \varepsilon]\big) < \infty$ for all $\varepsilon > 0$.

We set $\mathcal{S}^{\rm fin}_{\rm ord} := \{\mathcal{X} \in \mathcal{S}_{\rm ord} : \mathcal{X}^\pm(J) \text{ are finite}\}$.

Restricted models

Assume $\lambda_-(J_+), \lambda_+(J_-) > 0$. Then the following statements are equivalent:

(i) The model is positive recurrent, i.e., starting with an empty order book, we return to the empty order book in finite expected time.

(ii) There exists an invariant law on $\mathcal{S}^{\rm fin}_{\rm ord}$.

(iii) There exists an invariant law ν on $\mathcal{S}^{\rm fin}_{\rm ord}$ and the process is ergodic, in the sense that the law at time t of the process started in any finite initial state converges in total variation norm to ν as $t \to \infty$.

Luckock's equation

[Luckock '03] Let $M^{\pm}$ denote the price of the best buy/sell offer. Assume that the process is in equilibrium. Then

$$f_-(x) := \mathbb{P}[M^- < x] \quad \text{and} \quad f_+(x) := \mathbb{P}[M^+ > x]$$

solve the differential equation

$$\begin{aligned} &\text{(i)} \quad f_- \mathrm{d}\lambda_+ = -\lambda_- \mathrm{d}f_+, \\ &\text{(ii)} \quad f_+ \mathrm{d}\lambda_- = -\lambda_+ \mathrm{d}f_- \\ &\text{(iii)} \quad f_+(J_-) = 1 = f_-(J_+). \end{aligned}$$

Proof: Since buy orders are added to $A \subset J$ at the same rate as they are removed,

$$\int_A \mathbb{P}[M^- < x]\, \mathrm{d}\lambda_+(\mathrm{d}x) = \int_A \lambda_-(x)\, \mathbb{P}[M^+ \in \mathrm{d}x].$$

Luckock's equation

Assume $\lambda_-(J_+), \lambda_+(J_-) > 0$. Then Luckock's equation has a unique solution (f_-, f_+). In general, however, $f_\pm$ need not take values in $[0, 1]$. In such a case, by Luckock's result, no invariant law is possible.

Theorem [S. '16] Assume $\lambda_-(J_+), \lambda_+(J_-) > 0$. Then the Stigler-Luckock model on J is positive recurrent if and only if the unique solution to Luckock's equation satisfies $f_-(J_-) > 0$ and $f_+(J_+) > 0$.

Linear functionals

Let $\mathcal{X}_t^\pm$ denote the sets of buy and sell limit orders in the order book at time t and consider a weighted sum over the limit orders of the form

$$F(\mathcal{X}_t) := \int_J w_-(x)\mathcal{X}^-(\mathrm{d}x) + \int_J w_+(x)\mathcal{X}^+(\mathrm{d}x),$$

where $w_\pm : \overline{I} \to \mathbb{R}$ are "weight" functions.
Lemma One has

$$\tfrac{\partial}{\partial t}\mathbb{E}[F(\mathcal{X}_t)] = q_-(M_t^-) + q_+(M_t^+),$$

where $q_- : [J_-, J_+) \to \mathbb{R}$ and $q_+ : (J_+, J_-] \to \mathbb{R}$ are given by

$$\begin{aligned} q_-(x) &:= \int_x^{J_+} w_+ \mathrm{d}\lambda_+ - w_-(x)\lambda_+(x) \qquad \big(x \in [J_-, J_+)\big),\\ q_+(x) &:= -\int_{J_-}^{x} w_- \mathrm{d}\lambda_- - w_+(x)\lambda_-(x) \qquad \big(x \in (J_-, J_+]\big). \end{aligned}$$

Linear functionals

Proof of the theorem Assume $\lambda_-(J_+), \lambda_+(J_-) > 0$. For each $z \in \overline{J}$, there exist a unique pair of weight functions (w_-, w_+) such that

$$\tfrac{\partial}{\partial t}\mathbb{E}[F(\mathcal{X}_t)] = 1_{\{M_t^- \leq z\}} - f_-(z),$$

where (f_-, f_+) is the unique solution to Luckock's equation. Likewise, there exist a unique pair of weight functions (w_-, w_+) such that

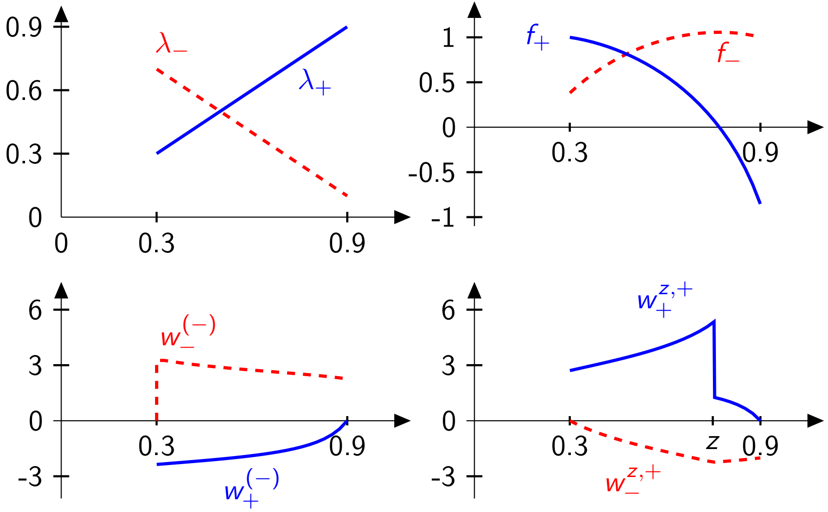
$$\tfrac{\partial}{\partial t}\mathbb{E}[F(\mathcal{X}_t)] = 1_{\{M_t^+ \geq z\}} - f_+(z).$$

In particular, there exist linear functionals $F^{(\pm)}$ such that

$$\begin{aligned}\tfrac{\partial}{\partial t}\mathbb{E}[F^{(-)}(\mathcal{X}_t)] &= 1_{\{M_t^- \leq J_-\}} - f_-(J_-),\\ \tfrac{\partial}{\partial t}\mathbb{E}[F^{(+)}(\mathcal{X}_t)] &= 1_{\{M_t^+ \geq J_+\}} - f_+(J_+).\end{aligned}$$

If $f_-(J_-), f_+(J_+) > 0$, then it is possible to construct a Lyapunov function from $F^{(-)}$ and $F^{(+)}$, proving positive recurrence. ∎

Restricted models



Restricted models

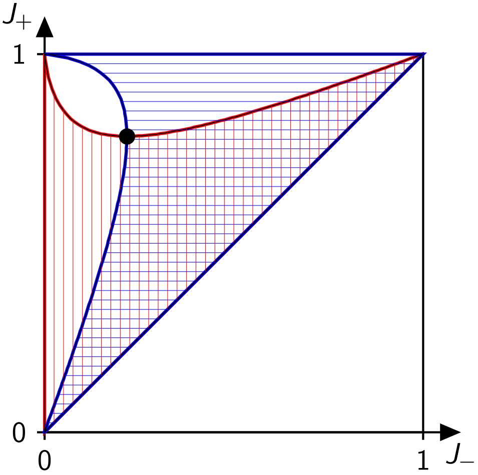
Define

$$\Lambda_-(J_-,J_+) := \frac{1}{\lambda_-(J_-)\lambda_-(J_+)} - \int_{J_-}^{J_+} \frac{1}{\lambda_+} \mathrm{d}\Big(\frac{1}{\lambda_-}\Big),$$
$$\Lambda_+(J_-,J_+) + := \frac{1}{\lambda_+(J_-)\lambda_+(J_+)} + \int_{J_-}^{J_+} \frac{1}{\lambda_-} \mathrm{d}\Big(\frac{1}{\lambda_+}\Big).$$

Then, for the restricted model on J,

$$f_-(J_-) > 0 \quad \Leftrightarrow \quad \Lambda_-(J_-,J_+) > 0,$$
$$f_+(J_+) > 0 \quad \Leftrightarrow \quad \Lambda_+(J_-,J_+) > 0.$$

Restricted models



Restrictions of the uniform model: subintervals (J_-, J_+) for which $f_-(J_-) > 0$ resp. $f_+(J_+) > 0$.

The critical window

The dot in the previous picture indicates the *critical window*, i.e., the unique subinterval J such that the solution of Luckock's equation on J satisfies $f_-(I_-) = 0 = f_+(I_+)$.

Recall that $V_{\rm W}$ is the Walrasian volume or trade. Let $V_{\max} := \lambda_-(I_-) \wedge \lambda_+(I_+)$. For $V \in [V_{\rm W}, V_{\max}]$, define

$$\begin{aligned} j_-(V) &= \sup\big\{x \in \bar{I} : \lambda_-(x) \geq V\big\}, \\ j_+(V) &= \inf\big\{x \in \bar{I} : \lambda_+(x) \geq V\big\}, \end{aligned}$$

and define $\Psi : [V_{\rm W}, V_{\max}] \to [-\infty, \infty)$ by

$$\Psi(V) := \frac{1}{V_{\rm W}^2} + \int_{V_{\rm W}}^{V} \Big\{\frac{1}{\lambda_+\big(j_-(W)\big)} + \frac{1}{\lambda_-\big(j_+(W)\big)}\Big\} \mathrm{d}\Big(\frac{1}{W}\Big).$$

The critical window

Luckock's volume of trade is given by

$$V_{\mathrm{L}} := \sup\big\{V \in [V_{\mathrm{W}}, V_{\max}] : \Psi(V) \geq 0\big\}.$$

Set $J = (J_-, J_+) := \big(j_-(V_{\mathrm{L}}), j_-(V_{\mathrm{L}})\big)$. If the Stigler-Luckock model has a critical window, then it is J. Conversely, if $\Psi(V_{\mathrm{L}}) = 0$ and $\overline{J} \subset I$, then J is a critical window.

For the uniform model, $V_{\mathrm{L}} = 1/z$ with z the unique solution of the equation $e^{-z} - z + 1 = 0$. Numerically, $V_{\mathrm{L}} \approx 0.78218829428020$.

Open problem:

- Show that a Stigler-Luckock model has an invariant law if and only if there exists a solution to Luckock's equation with $f_-(I_-), f_+(I_+) \geq 0$.

New progress

Frank Kelly and Elena Yudovina. *A Markov model of a limit order book: thresholds, recurrence, and trading strategies.* Preprint (2015 and 2016) ArXiv 1504.00579.

- **Claim** For any finite initial state, $\liminf_{t\to\infty} M_t^- = J_-$ and $\limsup_{t\to\infty} M_t^+ = J_+$ a.s.

A nice argument based on Kolmogorov's 0-1 law shows that the liminf and limsup are given by deterministic constants. The proof that they coincide with the boundary points of the critical window is rather more involved and needs additional technical assumtions. (In particular, $c\mathrm{d}x \leq \mu_\pm \leq C\mathrm{d}x$ for some $0 < c < C < \infty$.) Proofs use "fluid limits".

Market makers

The Stigler-Luckock model is unrealistic because of its huge spread. In reality, this attracts *market makers* who make money from buying for a low price and selling for a higher price.

We extend the model as follows: Apart from the buyers and sellers as before, with rate $\rho \geq 0$, a market maker arrives who places both a buy and a sell order, at the prices of the current bid and ask prices, respectively.
(If there are currently no buy (sell) limit orders in the order book, then the market maker does not place a buy (sell) limit order.)

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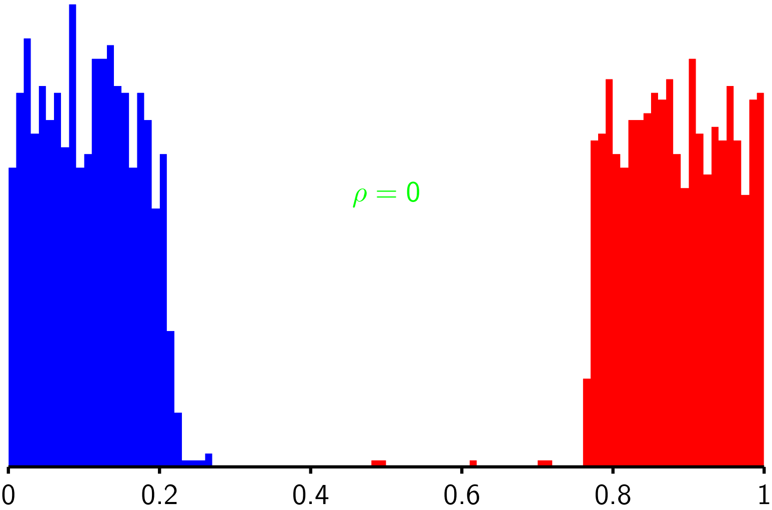

The order book after the arrival of 10,000 traders.

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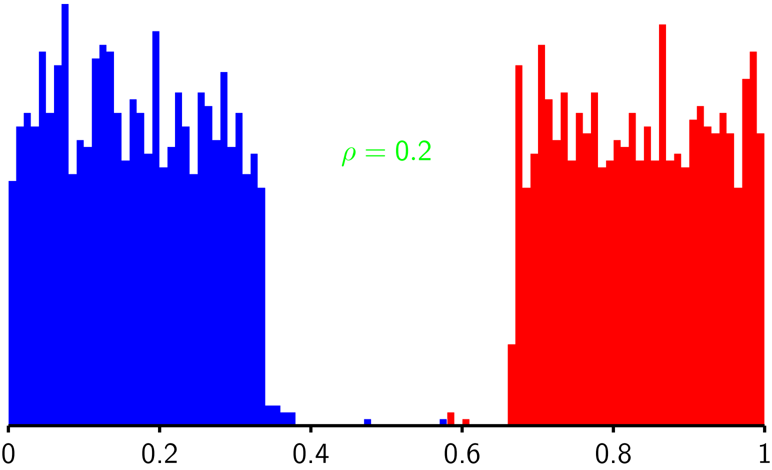

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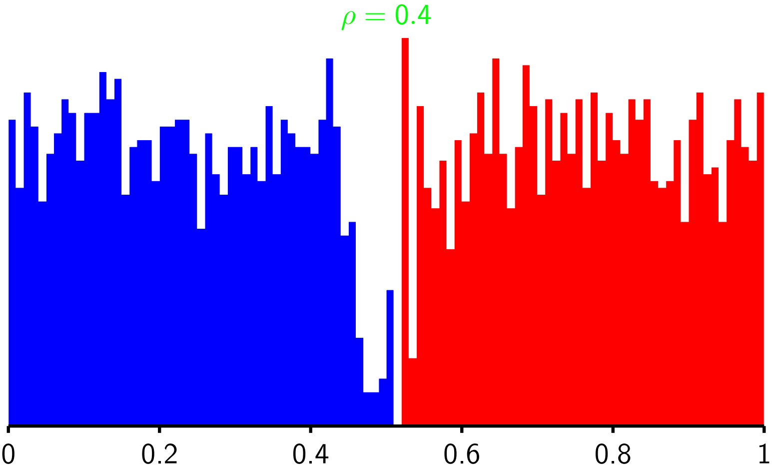

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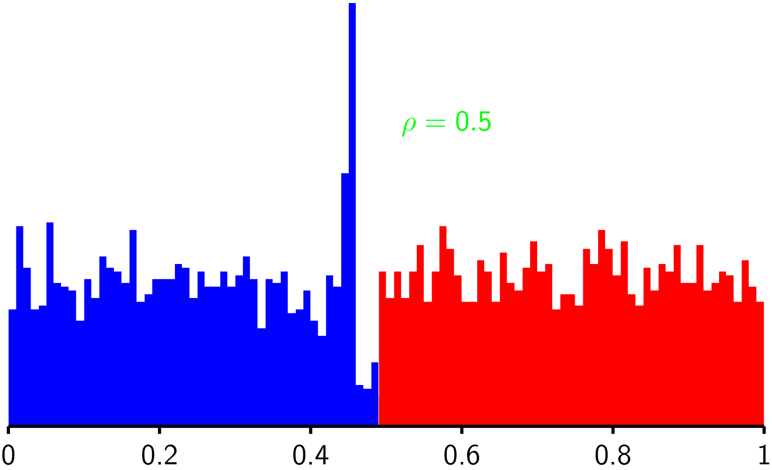

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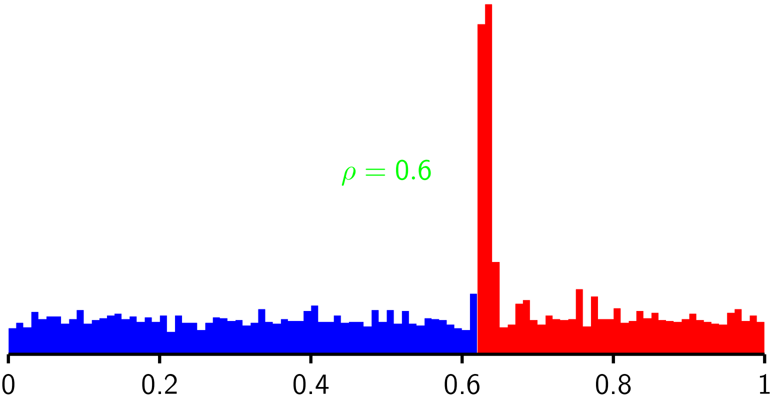

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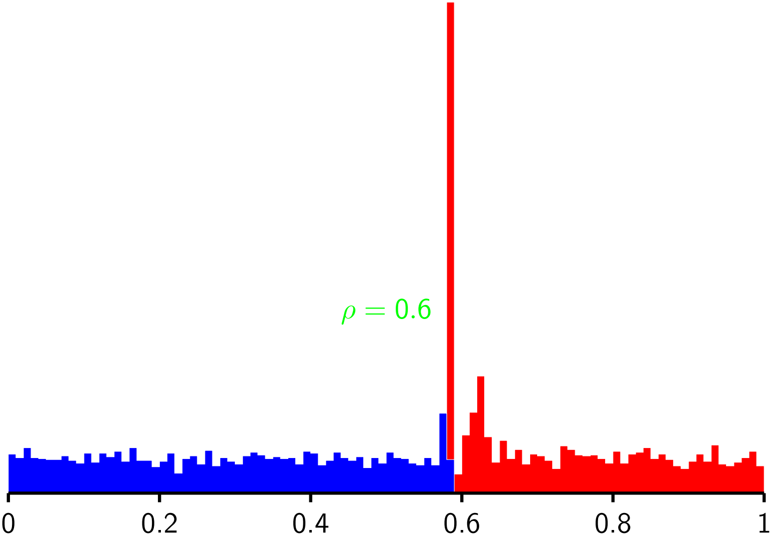

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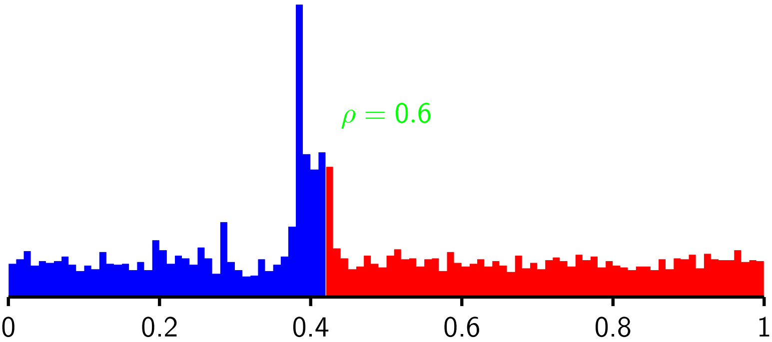

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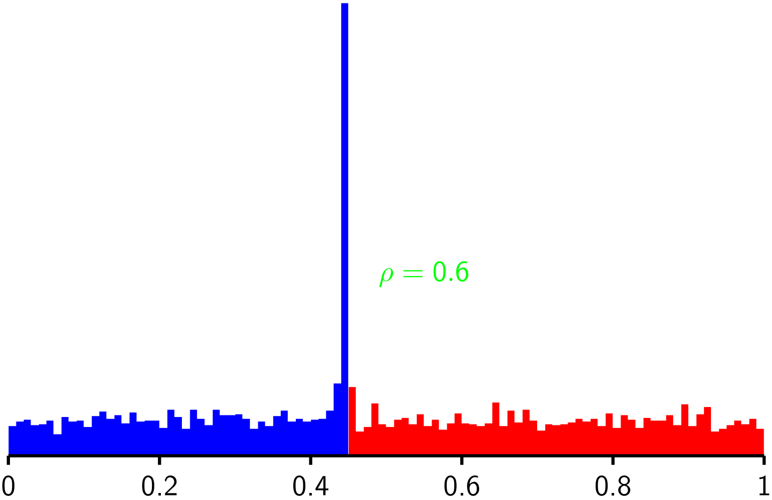

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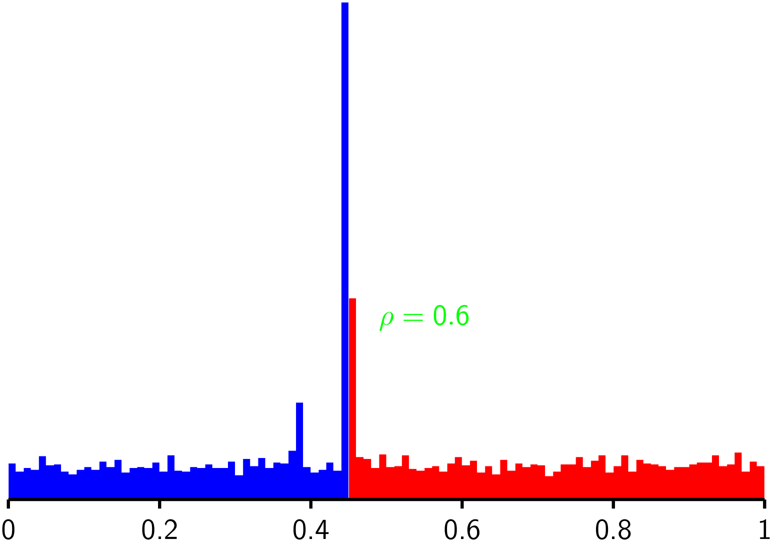

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Market makers

As long as $\rho < x_{\mathrm{W}}$ (with ρ the rate of market makers and x_{W} the Walrasian volume of trade), the model still has a critical window of positive length.

As soon as $\rho = x_{\mathrm{W}}$, the critical window closes.

When $\rho > x_{\mathrm{W}}$, we (numerically) observe "freezing", i.e., the price settles at a random level that is determined by the history of the process.

Future work

There is a lot of room for future work.

On the one hand, many phenomena observed in the existing models ask for a mathematical proof.

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- making the rate of market makers depend on the size of the spread
- better strategies for buyers, sellers, and market makers
- making the supply and demand functions depend on time

Future work

A little warning: less is often more.

Before you write down the most general possible model, keep in mind that for understanding the basic principles that are at work, it is often much more useful to have a minimal working example than a model with lots of parameters.

Today we have learned that even if buyers and sellers are in one place, there is still room for market makers who make money by transporting goods not in space but in time, buying when the price is low and selling when the price is high.

And in fact, you need these people
to attain Walras' equilibrium price.