Two applications of Bayesian networks

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Contents:

- Bayesian networks and the tasks solved with them
- Building “good” strategies using Bayesian network models
- Application 1: Adaptive testing
- Application 2: Decision-theoretic troubleshooting

Bayesian network

- a directed acyclic graph $G = (V, E)$
- each node $i \in V$ corresponds one random variable X_i with a finite set $\mathbb{X}_i$ of mutually exclusive states
- $pa(i)$ denotes the set of parents of node i in graph G
- to each node $i \in V$ corresponds a conditional probability table $P(X_i \mid (X_j)_{j \in pa(i)})$
- the DAG implies conditional independence relations between $(X_i)_{i \in V}$
- d-separation (Pearl, 1986) can be used to read the CI relations from the DAG

Using the chain rule we have that:

$$P((X_i)_{i \in V}) \quad = \quad \prod_{i \in V} P(X_i \mid X_{i-1}, \ldots, X_1)$$

Assume an ordering of $X_i, i \in V$ such that if $j \in pa(i)$ then $j < i$.
From the DAG we can read conditional independence relations

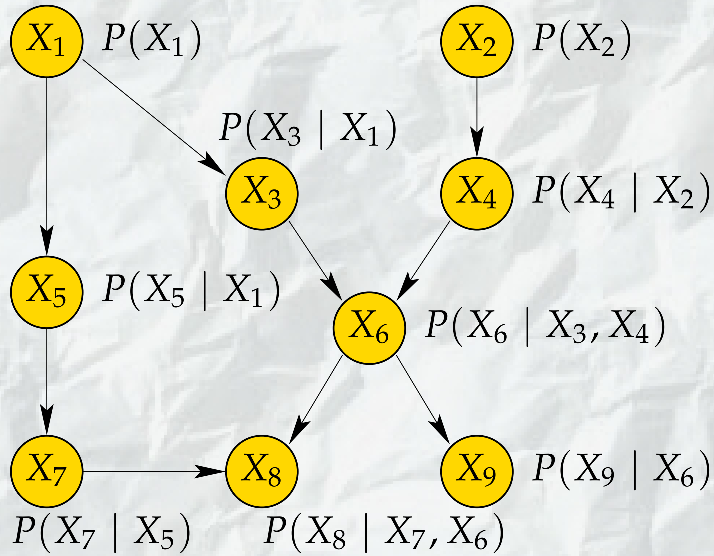
$$X_i \perp\!\!\!\perp X_k \mid (X_j)_{j \in pa(i)} \qquad \text{for } i \in V \text{ and } k < i \text{ and } k \notin pa(i)$$

Using the conditional independence relations from the DAG we get

$$P((X_i)_{i \in V}) \quad = \quad \prod_{i \in V} P(X_i \mid (X_j)_{j \in pa(i)}) \ .$$

It is the joint probability distribution represented by the Bayesian network.

Example:



$$
\begin{aligned}
P(X_1, \ldots, X_9) &= \\
&= P(X_9|X_8, \ldots, X_1) \cdot P(X_8|X_7, \ldots, X_1) \cdot \ldots \cdot P(X_2|X_1) \cdot P(X_1) \\
&= P(X_9|X_6) \cdot P(X_8|X_7, X_6) \cdot P(X_7|X_5) \cdot P(X_6|X_4, X_3) \\
&\quad \cdot P(X_5|X_1) \cdot P(X_4|X_2) \cdot P(X_3|X_1) \cdot P(X_2) \cdot P(X_1)
\end{aligned}
$$

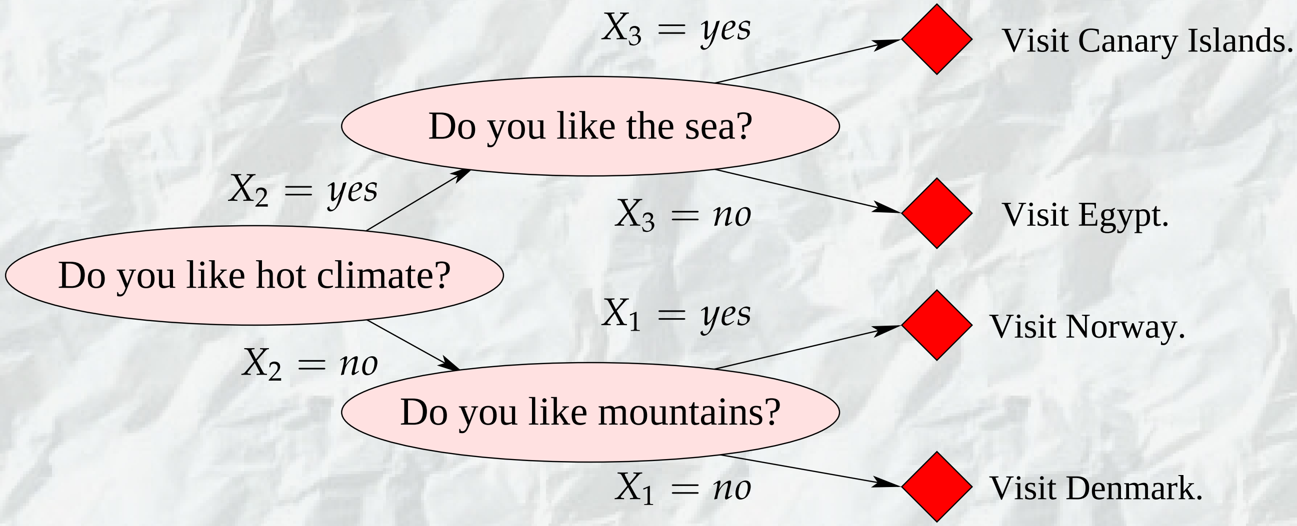
Typical use of Bayesian networks

- to model and explain a domain.
- to update beliefs about states of certain variables when some other variables were observed, i.e., computing conditional probability distributions, e.g., $P(X_{23}|X_{17} = yes, X_{54} = no)$.
- to find most probable configurations of variables
- to support decision making under uncertainty
- to find good strategies for solving tasks in a domain with uncertainty.

`Example 1: asia.net`

`Example 2: mildew4.net`

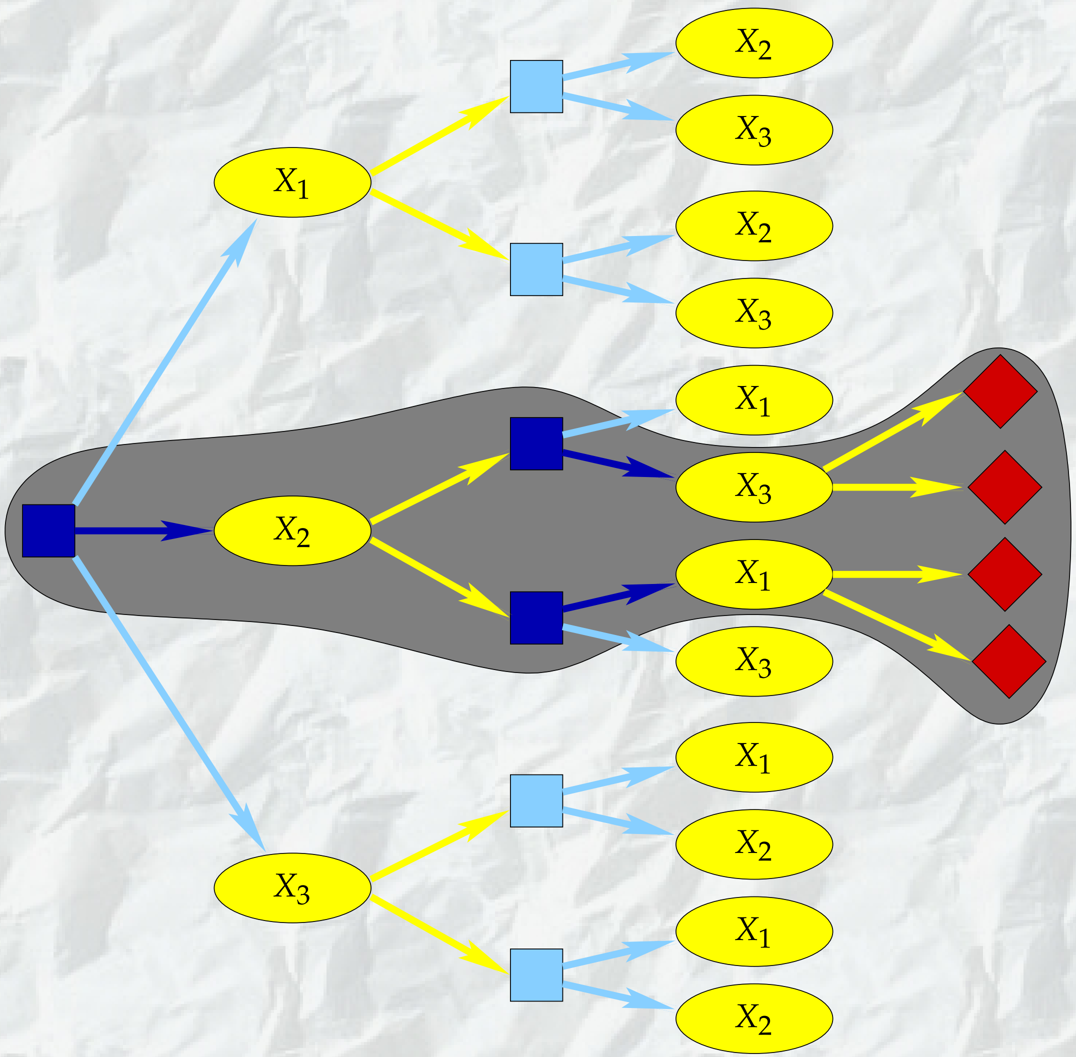
Strategy: “Holiday advisor”



For all nodes n of a strategy $\mathbf{s}$ we have defined:

- evidence $\mathbf{e}_n$, i.e. outcomes of steps performed to get to node n,
- probability $P(\mathbf{e}_n)$ of getting to node n, and
- utility $f(\mathbf{e}_n)$ being a real number.

Let $\mathcal{L}(\mathbf{s})$ be the set of terminal nodes of strategy $\mathbf{s}$.
Expected utility of strategy is $E_f(\mathbf{s}) = \sum_{\ell \in \mathcal{L}(\mathbf{s})} P(\mathbf{e}_\ell) \cdot f(\mathbf{e}_\ell)$.



Strategy $\mathbf{s}^\star$ is **optimal** iff it maximizes its expected utility.

Strategy $\mathbf{s}$ is **myopically optimal** iff each step of strategy $\mathbf{s}$ is selected so that it maximizes expected utility after the selected step is performed (*one step look ahead*).

Application 1: Adaptive test of basic operations with fractions

Examples of tasks:

$$T_1: \quad \left(\frac{3}{4} \cdot \frac{5}{6}\right) - \frac{1}{8} = \frac{15}{24} - \frac{1}{8} = \frac{5}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$T_2: \quad \frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

$$T_3: \quad \frac{1}{4} \cdot 1\frac{1}{2} = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$$

$$T_4: \quad \left(\frac{1}{2} \cdot \frac{1}{2}\right) \cdot \left(\frac{1}{3} + \frac{1}{3}\right) = \frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} = \frac{1}{6} \; .$$

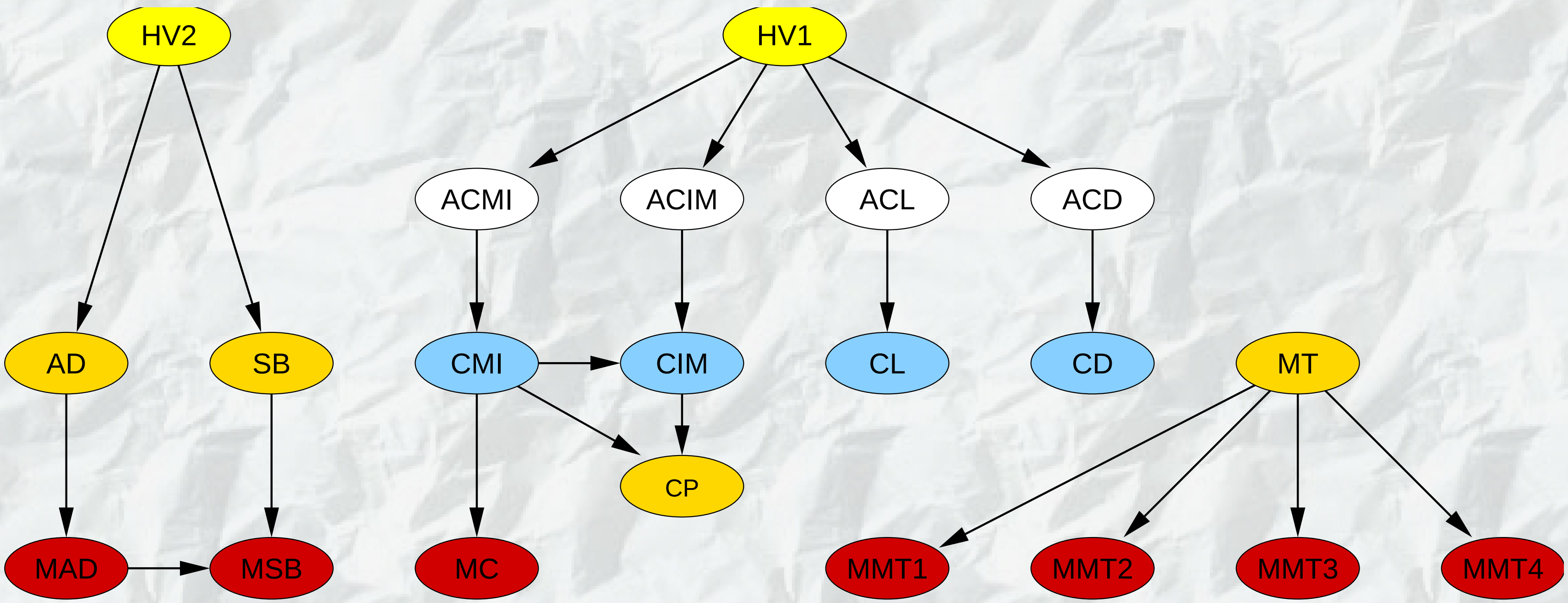
Elementary and operational skills

CP	Comparison (common numerator or denominator)	$\frac{1}{2} > \frac{1}{3}, \quad \frac{2}{3} > \frac{1}{3}$
AD	Addition (comm. denom.)	$\frac{1}{7} + \frac{2}{7} = \frac{1+2}{7} = \frac{3}{7}$
SB	Subtract. (comm. denom.)	$\frac{2}{5} - \frac{1}{5} = \frac{2-1}{5} = \frac{1}{5}$
MT	Multiplication	$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$
CD	Common denominator	$\left(\frac{1}{2}, \frac{2}{3}\right) = \left(\frac{3}{6}, \frac{4}{6}\right)$
CL	Cancelling out	$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}$
CIM	Conv. to mixed numbers	$\frac{7}{2} = \frac{3 \cdot 2 + 1}{2} = 3\frac{1}{2}$
CMI	Conv. to improp. fractions	$3\frac{1}{2} = \frac{3 \cdot 2 + 1}{2} = \frac{7}{2}$

Misconceptions

Label	Description	Occurrence
MAD	$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$	14.8%
MSB	$\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$	9.4%
MMT1	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a \cdot c}{b}$	14.1%
MMT2	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a+c}{b \cdot b}$	8.1%
MMT3	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$	15.4%
MMT4	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b+d}$	8.1%
MC	$a\frac{b}{c} = \frac{a \cdot b}{c}$	4.0%

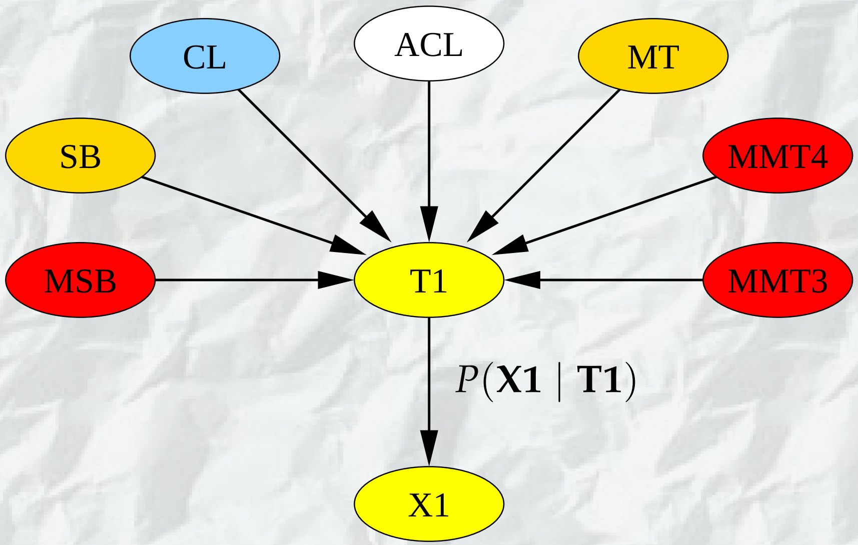
Student model



Evidence model for task $T1$

$$\left(\frac{3}{4} \cdot \frac{5}{6}\right) - \frac{1}{8} = \frac{15}{24} - \frac{1}{8} = \frac{5}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$T1 \quad \Leftrightarrow \quad MT \;\&\; CL \;\&\; ACL \;\&\; SB \;\&\; \neg MMT3 \;\&\; \neg MMT4 \;\&\; \neg MSB$$

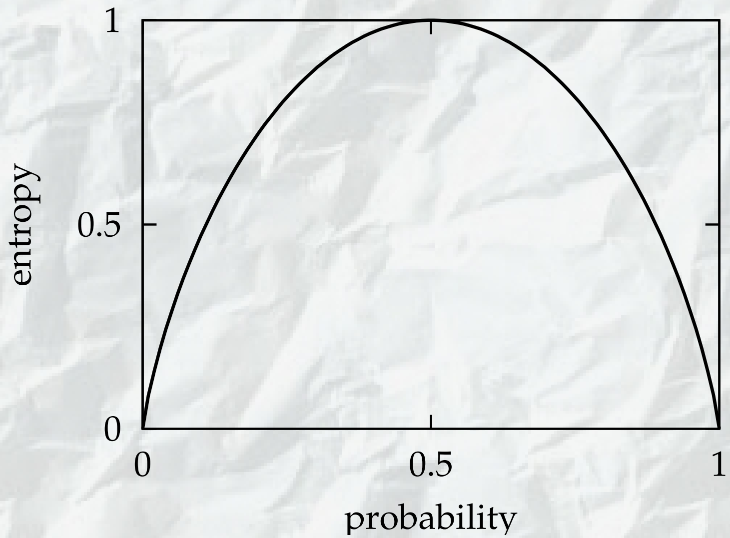


Hugin: `model-hv-2.net`

Using information gain as the utility function

"The lower the entropy of a probability distribution the more we know."

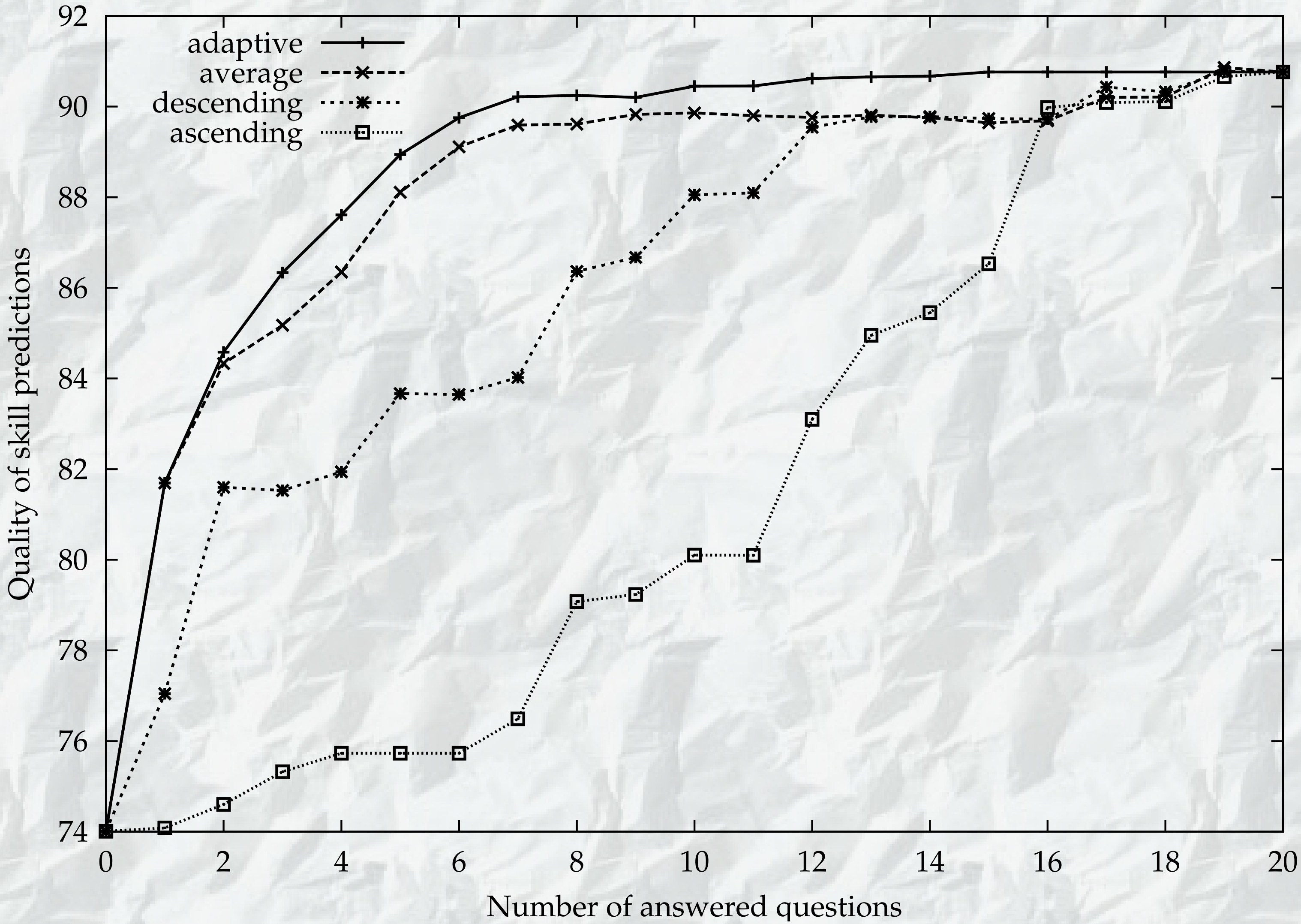
$$H\left(P(\mathbf{X})\right) = -\sum_{\mathbf{x}} P(\mathbf{X} = \mathbf{x}) \cdot \log P(\mathbf{X} = \mathbf{x})$$

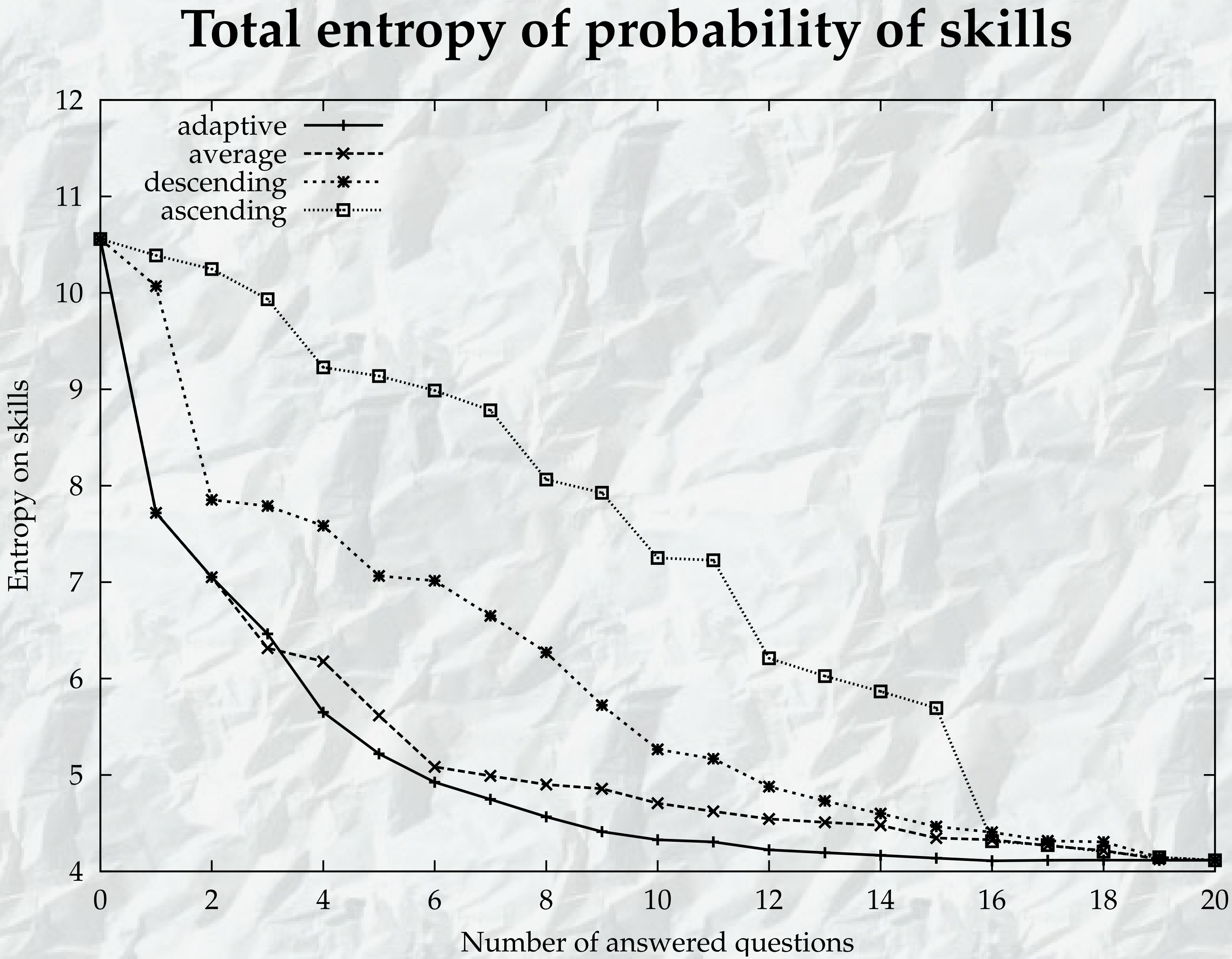


Information gain in a node n of a strategy

$$IG(\mathbf{e}_n) = H(P(\mathbf{S})) - H(P(\mathbf{S} \mid \mathbf{e}_n))$$

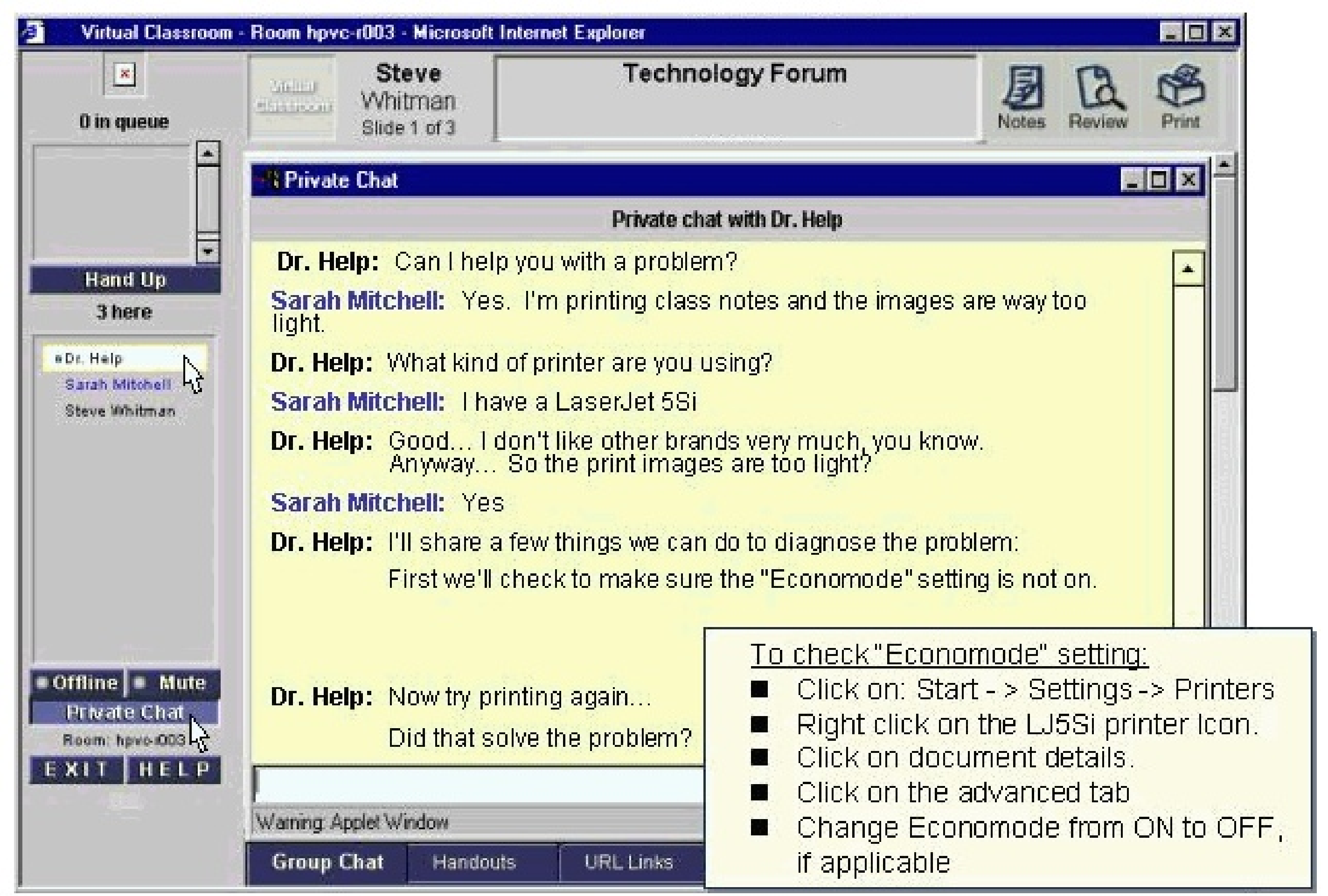
Skill Prediction Quality



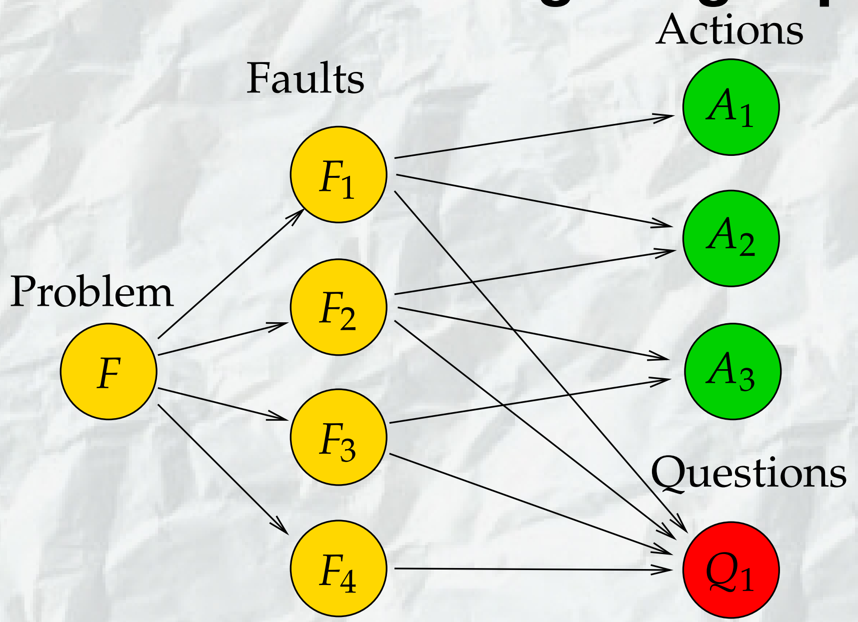


Total entropy of probability of skills
adaptive
average
descending
ascending
Entropy on skills
Number of answered questions
12
11
10
9
8
7
6
5
4
0
2
4
6
8
10
12
14
16
18
20

Application 2: Troubleshooting

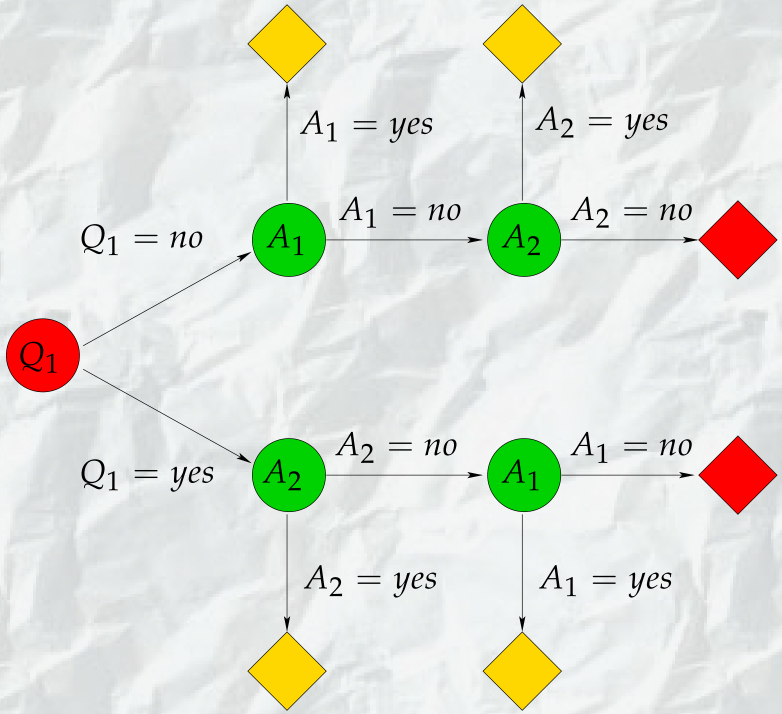


Application 2: Troubleshooting - Light print problem



- Problems: F_1 *Distribution problem*, F_2 *Defective toner*, F_3 *Corrupted dataflow*, and F_4 *Wrong driver setting*.
- Actions: A_1 *Remove, shake and reseat toner*, A_2 *Try another toner*, and A_3 *Cycle power*.
- Questions: Q_1 *Is the configuration page printed light?*

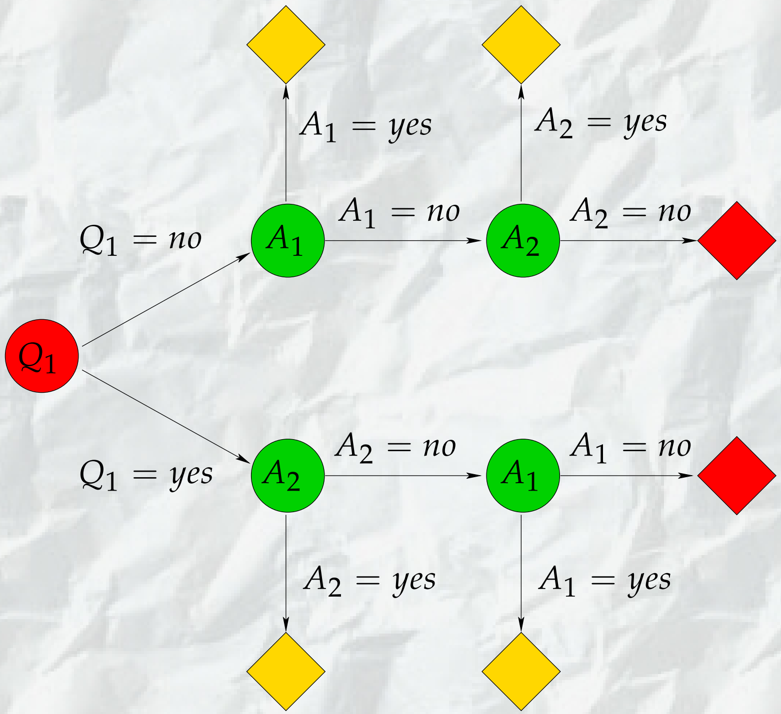
Troubleshooting strategy



The task is to find a strategy $\mathbf{s} \in \mathcal{S}$ minimising expected cost of repair

$$E_{CR}(\mathbf{s}) \quad = \quad \sum_{\ell \in \mathcal{L}(\mathbf{s})} P(\mathbf{e}_\ell) \cdot \left(\, t(\mathbf{e}_\ell) + c(\mathbf{e}_\ell) \, \right) \quad .$$

Expected cost of repair for a given strategy



$$
\begin{aligned}
E_{CR}(\mathbf{s}) \quad = \\
& P(Q_1 = no, A_1 = yes) \cdot (c_{Q_1} + c_{A_1}) \\
& +P(Q_1 = no, A_1 = no, A_2 = yes) \cdot (c_{Q_1} + c_{A_1} + c_{A_2}) \\
& +P(Q_1 = no, A_1 = no, A_2 = no) \cdot (c_{Q_1} + c_{A_1} + c_{A_2} + c_{CS}) \\
& +P(Q_1 = yes, A_2 = yes) \cdot (c_{Q_1} + c_{A_2}) \\
& +P(Q_1 = yes, A_2 = no, A_1 = yes) \cdot (c_{Q_1} + c_{A_2} + c_{A_1}) \\
& +P(Q_1 = yes, A_2 = no, A_1 = no) \cdot (c_{Q_1} + c_{A_2} + c_{A_1} + c_{CS})
\end{aligned}
$$

Demo: `headache_problem`

Commercial applications of Bayesian networks in educational testing and troubleshooting

- Hugin Expert A/S.
  software product: Hugin - a Bayesian network tool.
  http://www.hugin.com/
- Educational Testing Service (ETS)
  the world's largest private educational testing organization
  Research unit doing research on adaptive tests using Bayesian networks: http://www.ets.org/research/
- SACSO Project
  Systems for Automatic Customer Support Operations
  - research project of Hewlett Packard and Aalborg University.
  The troubleshooter offered as DezisionWorks by Dezide Ltd.
  http://www.dezide.com/