Applications of Bayesian networks

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Contents:

- Bayesian networks as a model for reasoning with uncertainty
- Building probabilistic models
- Building “good” strategies using the models
- Application 1: Adaptive testing
- Application 2: Decision-theoretic troubleshooting

Independence

If two discrete random variables are independent, the probability of the joint occurrence of values of two variables is equal to the product of the probabilities individually:

$$P(X = x, Y = y) \quad = \quad P(X = x) \cdot P(Y = y).$$

Also,

$$P(X = x | Y = y) = P(X = x)$$

- learning the value of Y does not influence your belief about X.

Example: `two_coins.net`

Conditional independence

If two variables are conditionally independent, the conditional probability of the joint occurrence given the value of another variable is equal to the product of the conditional probabilities:

$$P(X = x, Y = y|Z = z) \quad = \quad P(X = x|Z = z) \cdot P(Y = y|Z = z) \; .$$

- Also, learning the value of Z may influence your belief about X and about Y,
- but if you know the value of Z, learning the value of Y does not influence your belief about X.

$$P(X = x|Y = y, Z = z) \quad = \quad P(X = x|Z = z) \; .$$

Example: `two_biased_coins.net`

Pearl on Conditional independence (Pearl, 1988, p. 44)

- Conditional independence is not a grace of nature for which we must wait passively, but rather a psychological necessity which we satisfy actively by organizing our knowledge in a specific way.

- An important tool in such organization is the identification of intermediate variables that induce conditional independence among observables; if they are not in our vocabulary, we create them. *In medical diagnosis when some symptoms directly influence one another, the medical profession invents a name for that interaction (e.g., "syndrome", "complication," "pathological state") and treats it as a new auxiliary variable that induces conditional independence;*

- dependency between any two interacting systems is fully attributed to the dependencies of each on the auxiliary variable.

Building up complex networks

- Relationships among many variables are modeled in terms of important relationships among smaller subsets of variables.

Example: Wet grass on Holmes' lawn can be caused either by rain or by his sprinkler.

$$
\begin{aligned}
& P(Holmes, Watson, Rain, Sprinkler) \\
= \quad & P(Holm|Wat, Rn, Sprnk) \cdot P(Wat|Rn, Sprnk) \cdot P(Rn|Sprnk) \cdot P(Sprnk) \\
= \quad & P(Holm|Rn, Sprnk) \cdot P(Wat|Rn) \cdot P(Rn) \cdot P(Sprnk)
\end{aligned}
$$

Example: `wet_grass.net`

Building up complex Bayesian networks

- Acyclic directed graphs (DAGs):
- Nodes correspond to variables
- Directed edges represent explicit dependence relationships
- No edges means no explicit dependence, although there can be dependence through relationships with other variables.

`Example: asia.net`

Building Bayesian network models

three basic approaches

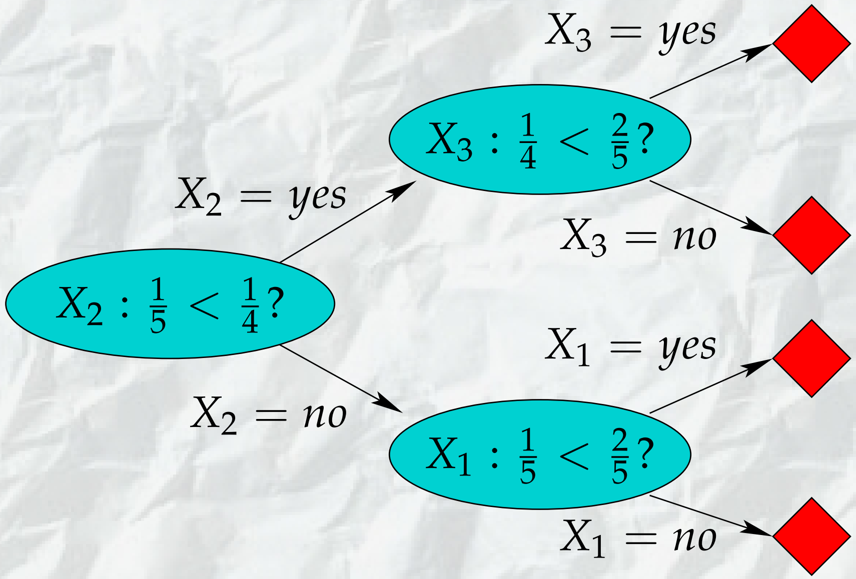
- Discussions with domain experts: expert knowledge is used to get the structure and parameters of the model
- A dataset of records is collected and a machine learning method is used to to construct a model and estimate its parameters.
- A combination of previous two: e.g. experts helps with the stucture, data are used to estimate parameters.

Typical tasks solved using Bayesian networks

Bayesian networks are used:

- to model and explain a domain.
- to update beliefs about states of certain variables when some other variables were observed, i.e., computing conditional probability distributions, e.g., $P(X_{23}|X_{17} = yes, X_{54} = no)$.
- to find most probable configurations of variables
- to support decision making under uncertainty
- to find good strategies for solving tasks in a domain with uncertainty.

Example of a strategy



X_3 is more difficult question than X_2 which is more difficult than X_1.

Building strategies using the models

For all terminal nodes $\ell \in \mathcal{L}(\mathbf{s})$ of a strategy $\mathbf{s}$ we have defined:

- steps that were performed to get to that node together with their outcomes. It is called collected evidence $\mathbf{e}_\ell$.
- Using the probabilistic model of the domain we can compute probability of getting to that terminal node $P(\mathbf{e}_\ell)$.

During the process of collecting evidence $\mathbf{e}$ we update the probability of getting to a terminal node, which corresponds to conditional probability $P(\mathbf{e}_\ell \mid \mathbf{e})$, where $\mathbf{e}$ is evidence collected as far.

Building strategies using the models

For all terminal nodes $\ell \in \mathcal{L}(\mathbf{s})$ of a strategy $\mathbf{s}$ we have also defined:

- an evaluation function $f : \cup_{\mathbf{s} \in \mathcal{S}} \mathcal{L}(\mathbf{s}) \mapsto \mathbb{R}$.

For each strategy we can compute:

- expected value of the strategy:

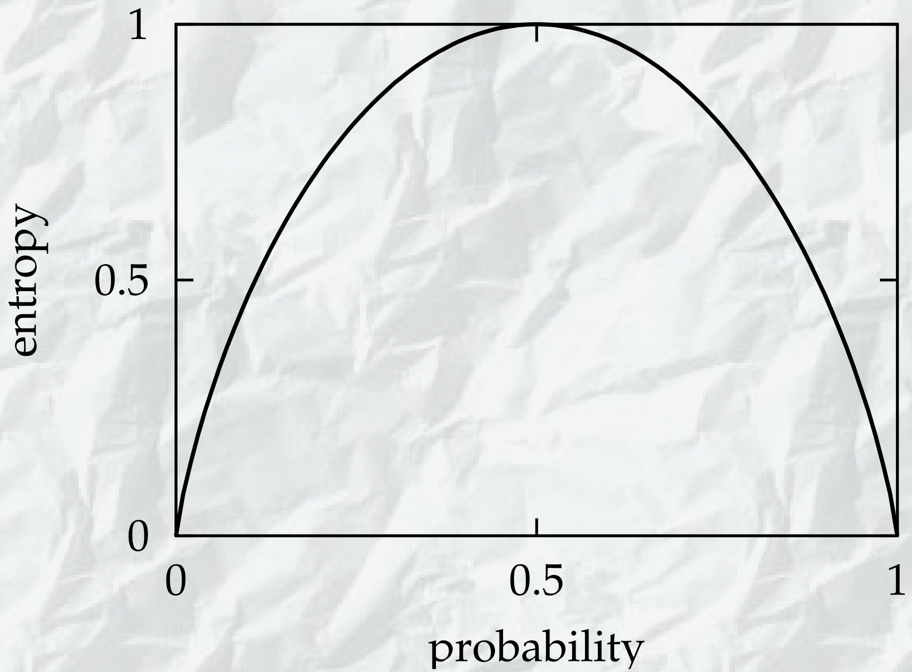
$$E_f(\mathbf{s}) = \sum_{\ell \in \mathcal{L}(\mathbf{s})} P(\mathbf{e}_\ell) \cdot f(\mathbf{e}_\ell)$$

The goal:

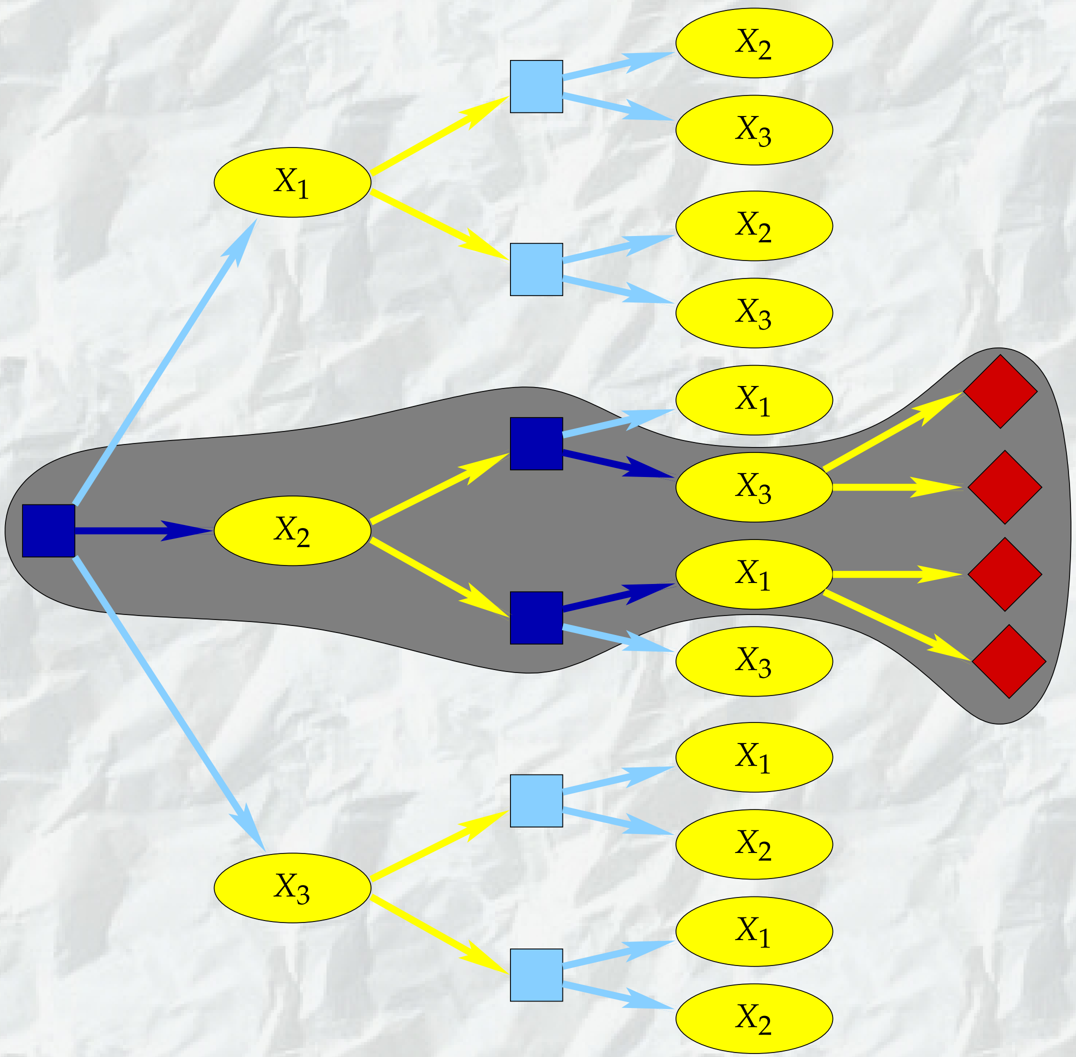
- find a strategy that maximizes (minimizes) its expected value

Using entropy as an information measure

"The lower the entropy of a probability distribution the more we know."



$$H\left(P(\mathbf{X})\right) = -\sum_{\mathbf{x}} P(\mathbf{X}=\mathbf{x}) \cdot \log P(\mathbf{X}=\mathbf{x})$$

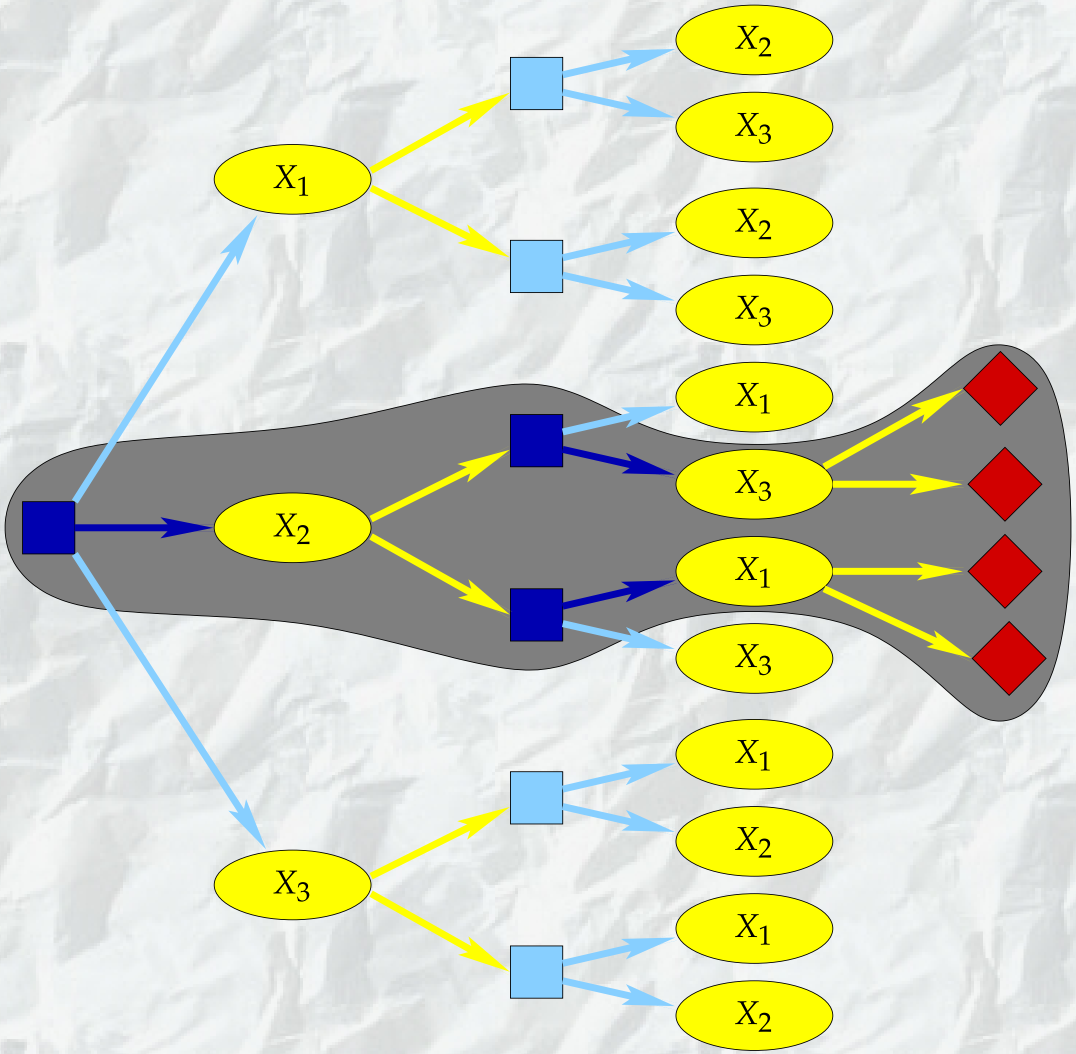


Entropy in node n

$$H(\mathbf{e}_n) = H(P(\mathbf{S} \mid \mathbf{e}_n))$$

Expected entropy at the end of test $\mathbf{t}$

$$E_H(\mathbf{t}) = \sum_{\ell \in \mathcal{L}(\mathbf{t})} P(\mathbf{e}_\ell) \cdot H(\mathbf{e}_\ell)$$



$\mathcal{T}$ ... the set of all possible tests

A test $\mathbf{t}^\star$ is **optimal** iff

$$\mathbf{t}^\star = \arg\min_{\mathbf{t}\in\mathcal{T}} E_H(\mathbf{t}) \ .$$

A test $\mathbf{t}$ is **myopically optimal** iff each question $X^\star$ of $\mathbf{t}$ minimizes the expected value of entropy after the question is answered:

$$X^\star = \arg\min_{X\in\mathcal{X}} E_H(\mathbf{t}_{\downarrow X}) \ ,$$

i.e. it works as if the test finished after the selected question $X^\star$.

Application 1: Adaptive test of basic operations with fractions

Examples of tasks:

$$T_1: \quad \left(\frac{3}{4}\cdot\frac{5}{6}\right)-\frac{1}{8} = \frac{15}{24}-\frac{1}{8}=\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

$$T_2: \quad \frac{1}{6}+\frac{1}{12} = \frac{2}{12}+\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$$

$$T_3: \quad \frac{1}{4}\cdot 1\frac{1}{2} = \frac{1}{4}\cdot\frac{3}{2}=\frac{3}{8}$$

$$T_4: \quad \left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot\left(\frac{1}{3}+\frac{1}{3}\right) = \frac{1}{4}\cdot\frac{2}{3}=\frac{2}{12}=\frac{1}{6}\ .$$

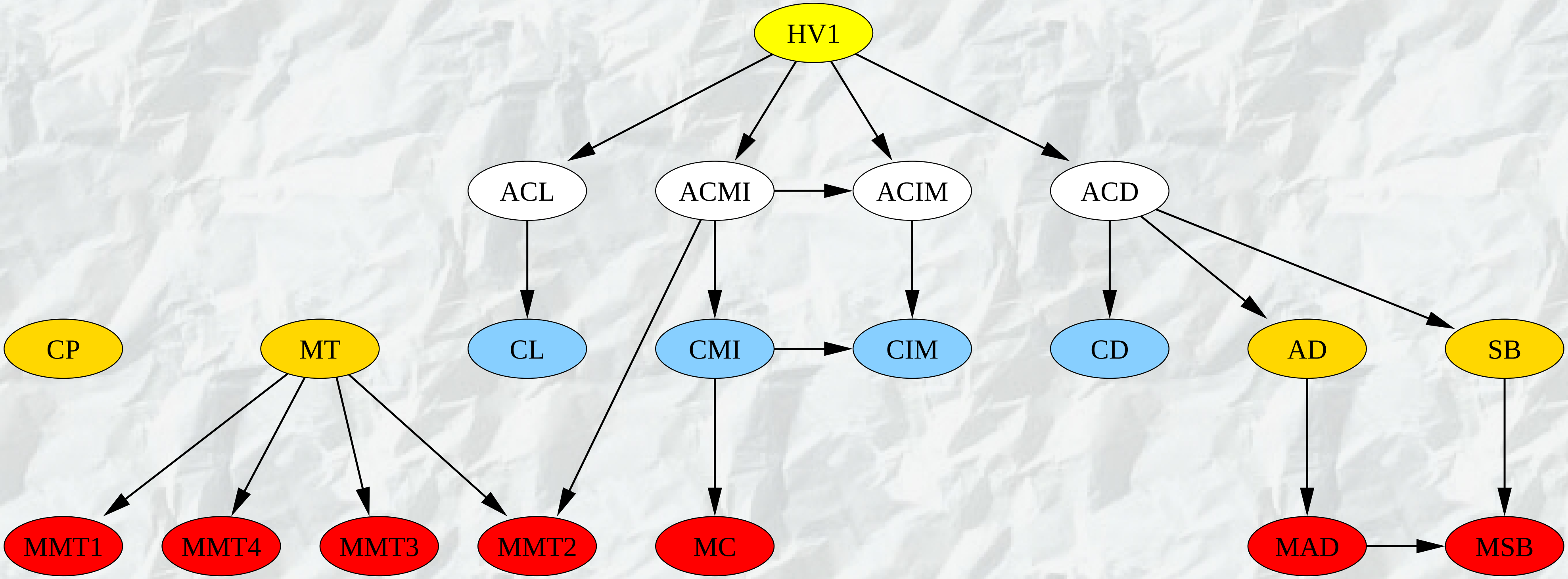
Elementary and operational skills

CP	Comparison (common numerator or denominator)	$\frac{1}{2} > \frac{1}{3}, \quad \frac{2}{3} > \frac{1}{3}$
AD	Addition (comm. denom.)	$\frac{1}{7} + \frac{2}{7} = \frac{1+2}{7} = \frac{3}{7}$
SB	Subtract. (comm. denom.)	$\frac{2}{5} - \frac{1}{5} = \frac{2-1}{5} = \frac{1}{5}$
MT	Multiplication	$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$
CD	Common denominator	$\left(\frac{1}{2}, \frac{2}{3}\right) = \left(\frac{3}{6}, \frac{4}{6}\right)$
CL	Cancelling out	$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}$
CIM	Conv. to mixed numbers	$\frac{7}{2} = \frac{3 \cdot 2+1}{2} = 3\frac{1}{2}$
CMI	Conv. to improp. fractions	$3\frac{1}{2} = \frac{3 \cdot 2+1}{2} = \frac{7}{2}$

Misconceptions

Label	Description	Occurrence
MAD	$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$	14.8%
MSB	$\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$	9.4%
MMT1	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a \cdot c}{b}$	14.1%
MMT2	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a+c}{b \cdot b}$	8.1%
MMT3	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$	15.4%
MMT4	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b+d}$	8.1%
MC	$a\frac{b}{c} = \frac{a \cdot b}{c}$	4.0%

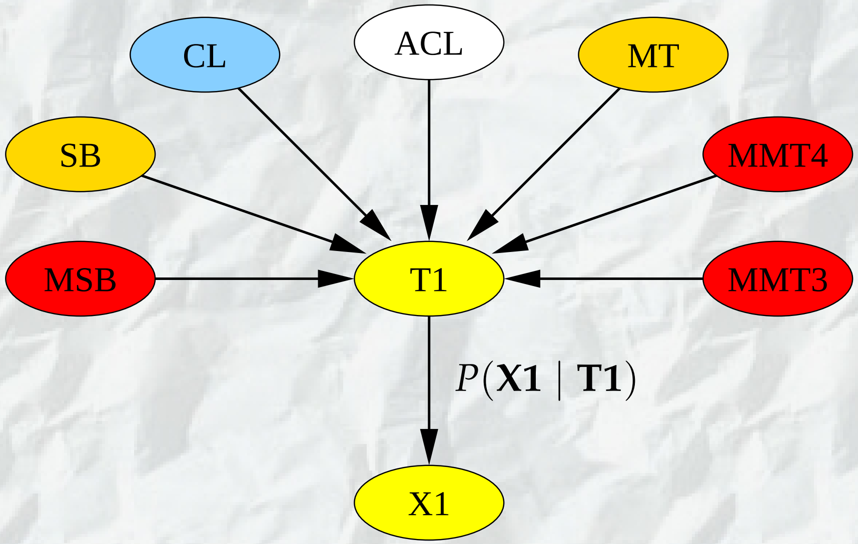
Student model



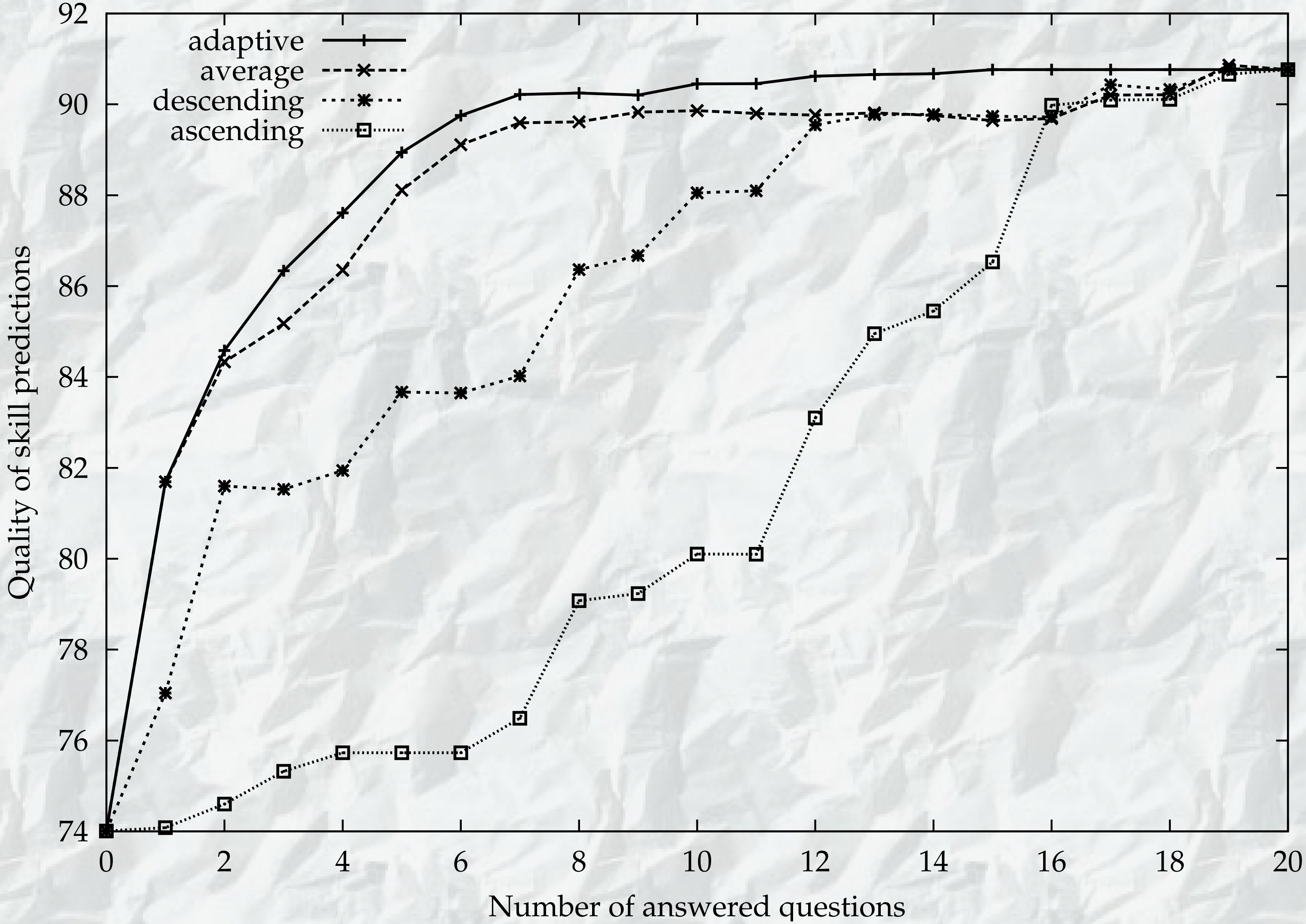
Evidence model for task $T1$

$$\left(\frac{3}{4}\cdot\frac{5}{6}\right)-\frac{1}{8}=\frac{15}{24}-\frac{1}{8}=\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

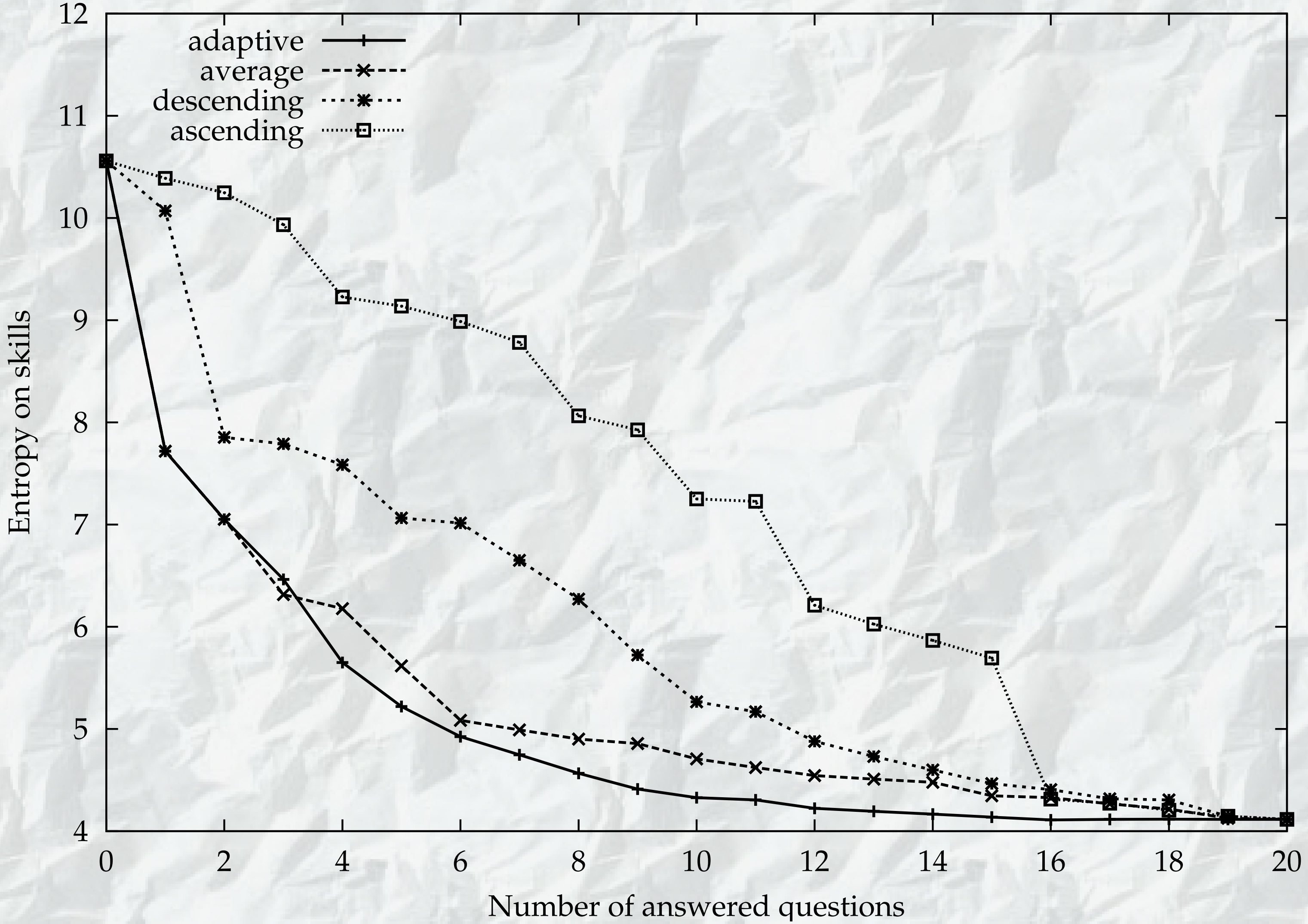
$$T1 \quad \Leftrightarrow \quad MT \,\&\, CL \,\&\, ACL \,\&\, SB \,\&\, \neg MMT3 \,\&\, \neg MMT4 \,\&\, \neg MSB$$



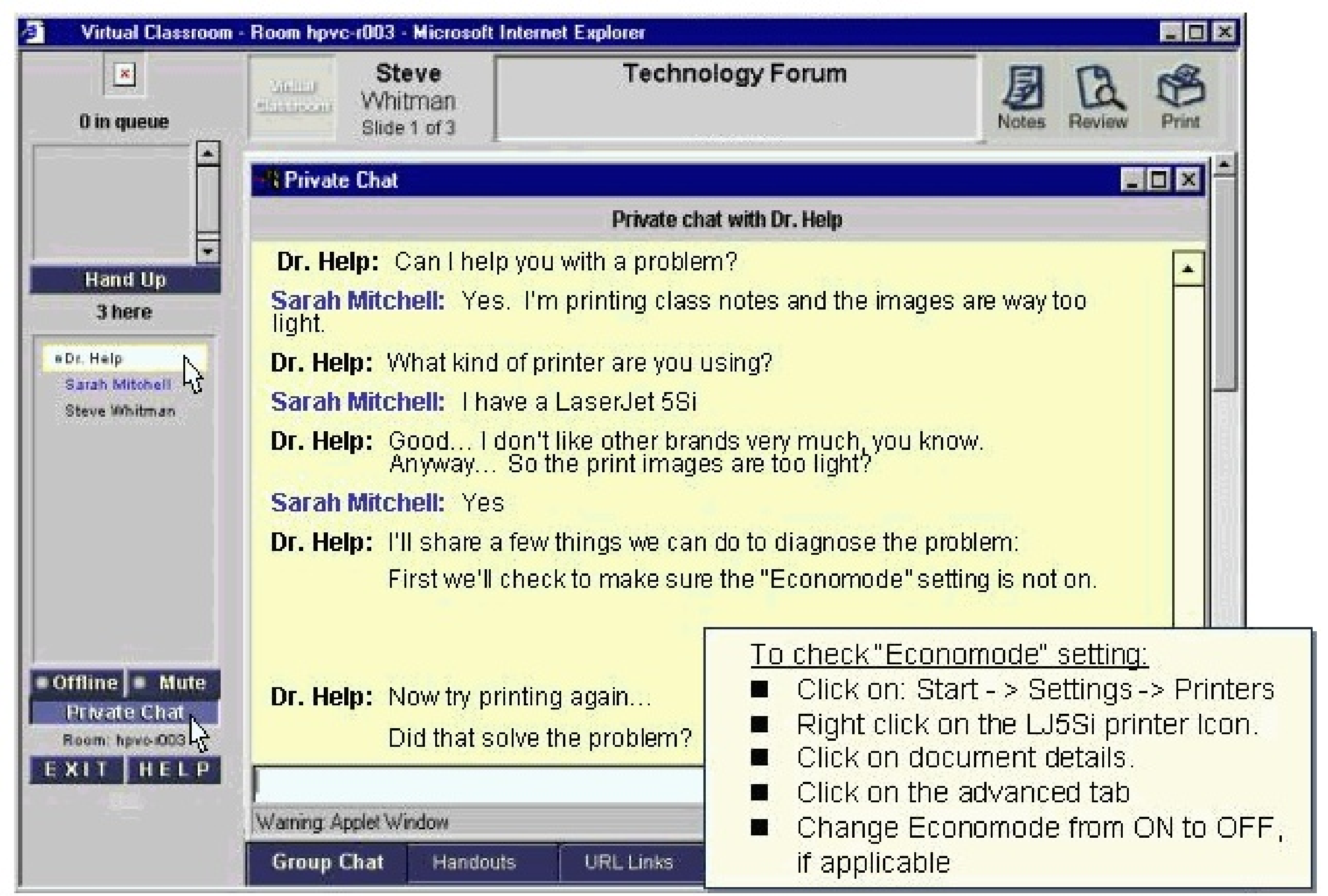
Skill Prediction Quality



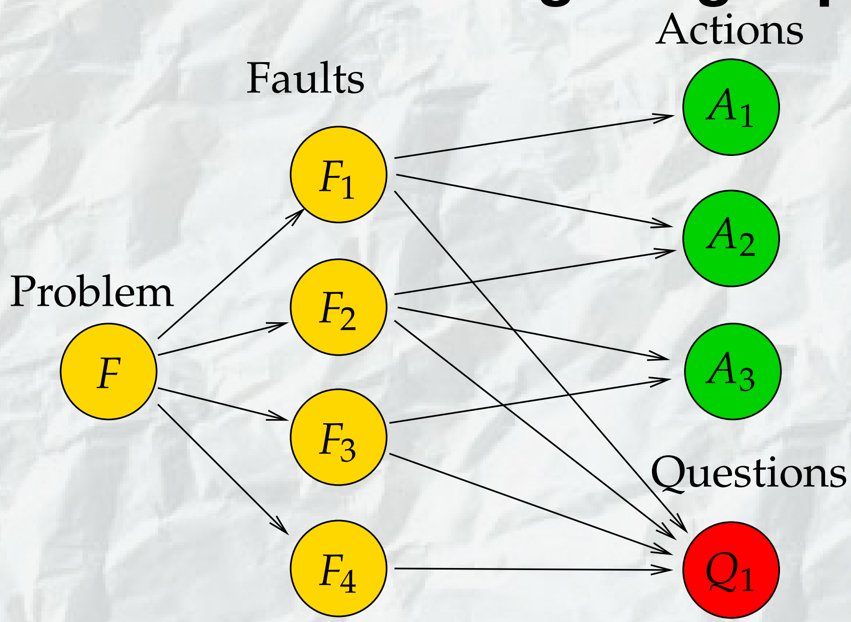
Total entropy of probability of skills



Application 2: Troubleshooting

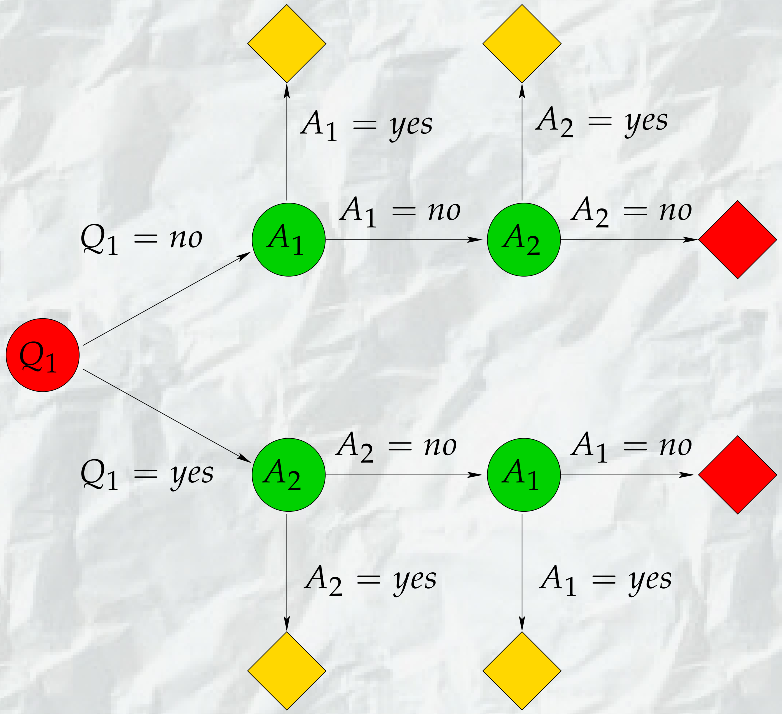


Application 2: Troubleshooting - Light print problem



- Problems: F_1 *Distribution problem*, F_2 *Defective toner*, F_3 *Corrupted dataflow*, and F_4 *Wrong driver setting*.
- Actions: A_1 *Remove, shake and reseat toner*, A_2 *Try another toner*, and A_3 *Cycle power*.
- Questions: Q_1 *Is the configuration page printed light?*

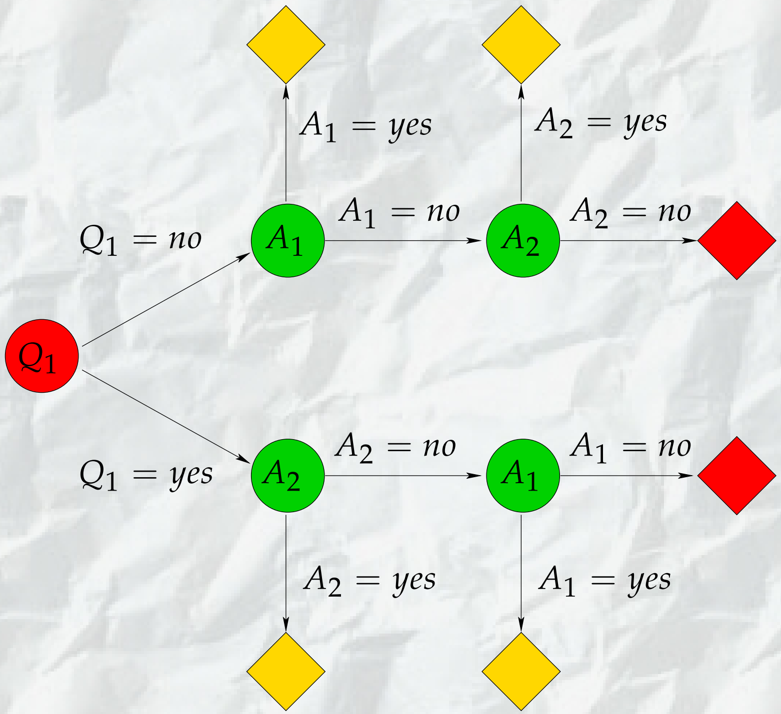
Troubleshooting strategy



The task is to find a strategy $\mathbf{s} \in \mathcal{S}$ minimising expected cost of repair

$$E_{CR}(\mathbf{s}) = \sum_{\ell \in \mathcal{L}(\mathbf{s})} P(\mathbf{e}_\ell) \cdot \left(t(\mathbf{e}_\ell) + c(\mathbf{e}_\ell) \right) .$$

Expected cost of repair for a given strategy



$$
\begin{aligned}
E_{CR}(\mathbf{s}) \quad = \\
& P(Q_1 = no, A_1 = yes) \cdot (c_{Q_1} + c_{A_1}) \\
& +P(Q_1 = no, A_1 = no, A_2 = yes) \cdot (c_{Q_1} + c_{A_1} + c_{A_2}) \\
& +P(Q_1 = no, A_1 = no, A_2 = no) \cdot (c_{Q_1} + c_{A_1} + c_{A_2} + c_{CS}) \\
& +P(Q_1 = yes, A_2 = yes) \cdot (c_{Q_1} + c_{A_2}) \\
& +P(Q_1 = yes, A_2 = no, A_1 = yes) \cdot (c_{Q_1} + c_{A_2} + c_{A_1}) \\
& +P(Q_1 = yes, A_2 = no, A_1 = no) \cdot (c_{Q_1} + c_{A_2} + c_{A_1} + c_{CS})
\end{aligned}
$$

Demo: `light_print_problem`

Commercial applications of Bayesian networks in educational testing and troubleshooting

- Hugin Expert A/S.
  software product: Hugin - a Bayesian network tool.
  http://www.hugin.com/
- Educational Testing Service (ETS)
  the world's largest private educational testing organization
  Research unit doing research on adaptive tests using Bayesian networks: http://www.ets.org/research/
- SACSO Project
  Systems for Automatic Customer Support Operations
  - research project of Hewlett Packard and Aalborg University.
  The troubleshooter offered as DezisionWorks by Dezide Ltd.
  http://www.dezide.com/

...and it is time to end.