Bayesian Networks in Educational Testing

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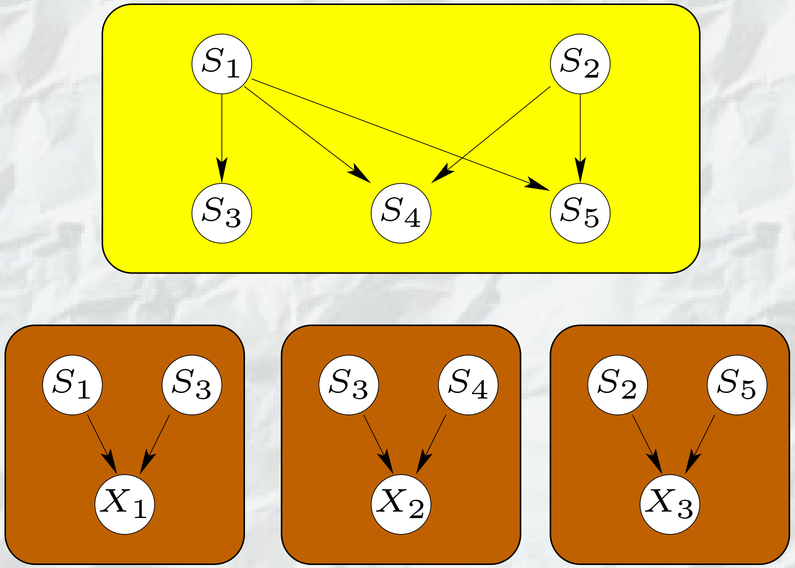
This presentation is available at:
http://www.utia.cas.cz/vomlel/slides/pgm02slides.pdf

Contents:

- Student model and evidence models.
  Knowledge about a student.
- Difference between a fixed test and an adaptive test.
- Optimal and myopically optimal tests.
- Construction of a myopically optimal fixed test.
- Test of basic operations with fractions.
- Results of experiments.

Student and evidence models

(R. Almond and R. Mislevy, 1999)

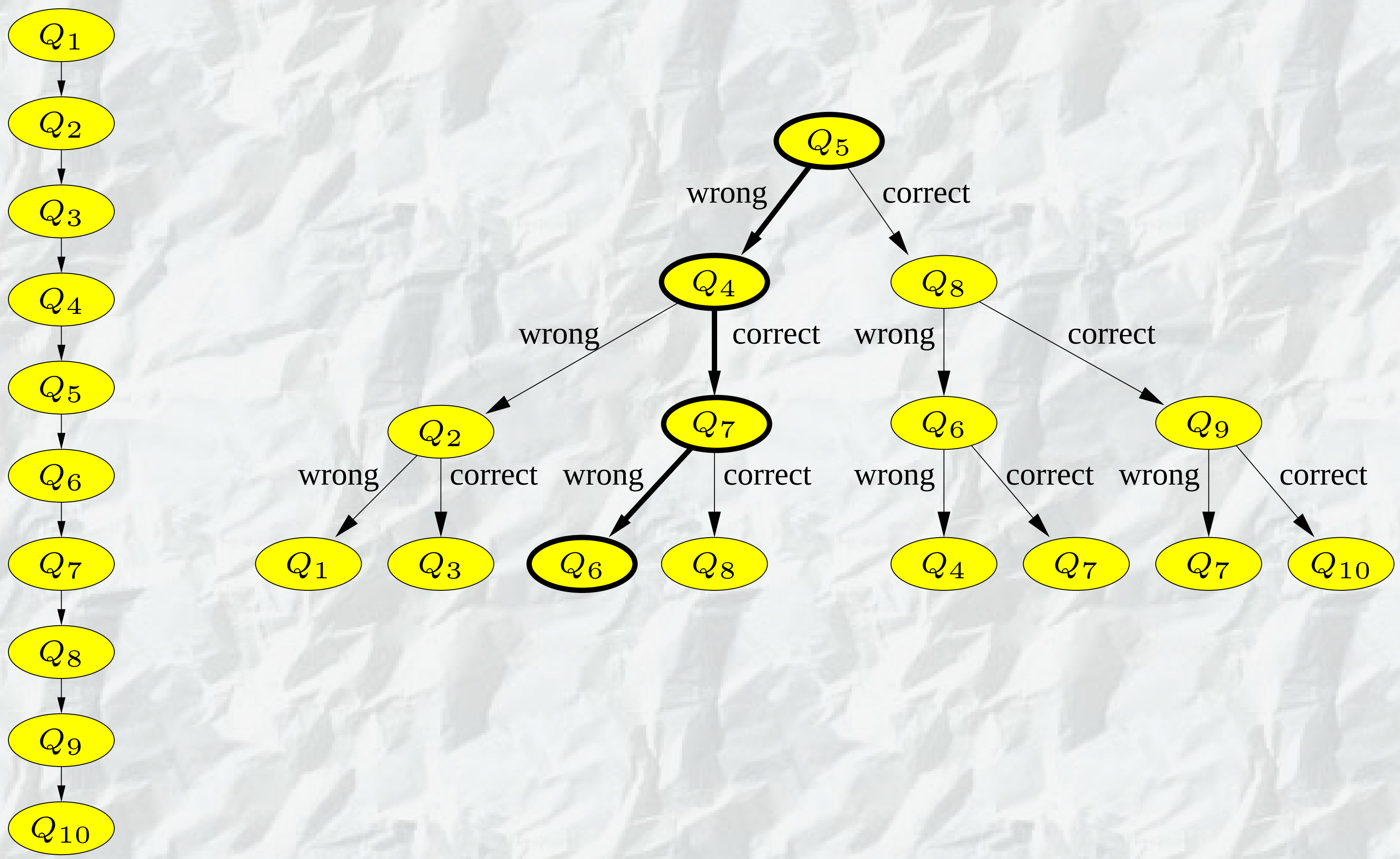


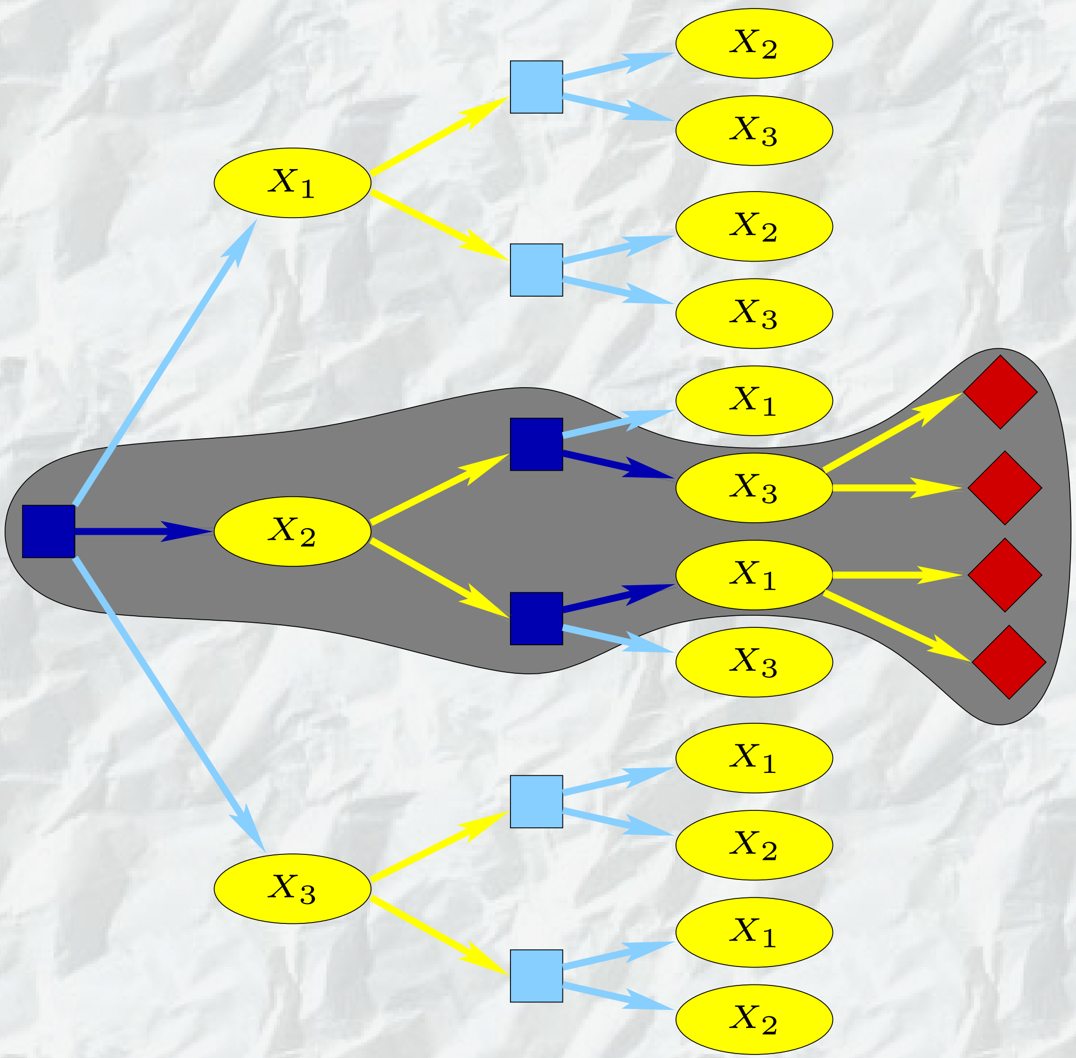
Entropy of probability distribution $P(\mathbf{S})$

$$H\left(P(\mathbf{S})\right) = -\sum_{\mathbf{s}} P(\mathbf{S}=\mathbf{s}) \cdot \log P(\mathbf{S}=\mathbf{s})$$

“The lower the entropy the more we know about a student.”

Fixed Test vs. Adaptive Test





Entropy in node n

$$H(\mathbf{e}_n) \quad = \quad H(P(\mathbf{S} \mid \mathbf{e}_n))$$

Expected entropy at the end of test $\mathbf{t}$

$$E_H(\mathbf{t}) \quad = \quad \sum_{\ell \in \mathcal{L}(\mathbf{t})} P(\mathbf{e}_\ell) \cdot H(\mathbf{e}_\ell)$$

$\mathcal{T}$ ... the set of all possible tests (e.g. of a given length)

A test $\mathbf{t}^\star$ is **optimal** iff

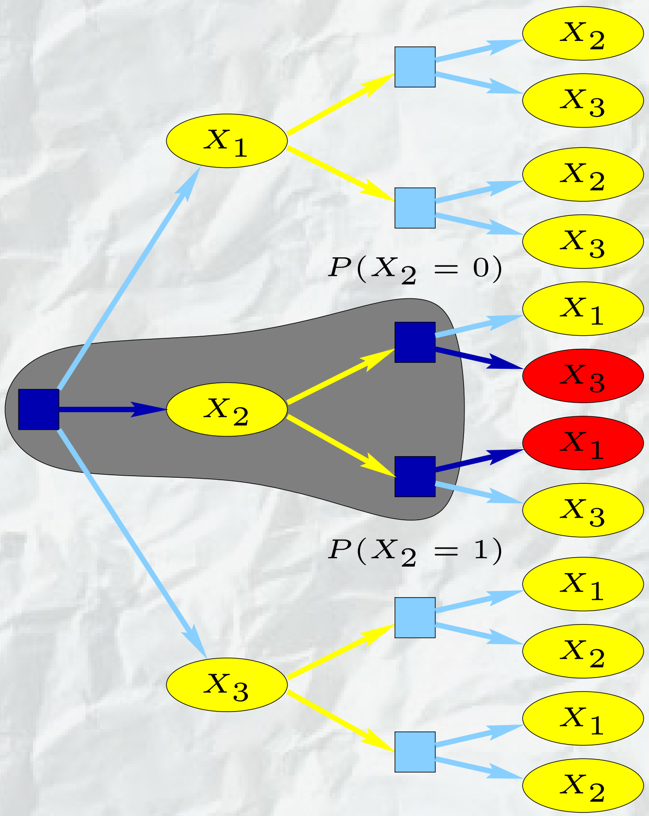
$$\mathbf{t}^\star \quad = \quad \arg\min_{\mathbf{t} \in \mathcal{T}} E_H(\mathbf{t}) \quad .$$

A **myopically optimal** test $\mathbf{t}$ is a test where each question $X^\star$ of $\mathbf{t}$ minimizes the expected value of entropy after the question is answered:

$$X^\star \quad = \quad \arg\min_{X \in \mathcal{X}} E_H(\mathbf{t}_{\downarrow X}) \quad ,$$

i.e. it works as if the test finished after the selected question $X^\star$.

Myopic construction of a fixed test



$$
\begin{aligned}
e\_list &= \{\{X_2 = 0\}, \{X_2 = 1\}\} \\
counts[3] &= P(X_2 = 0) \; = \; 0.7 \\
counts[1] &= P(X_2 = 1) \; = \; 0.3
\end{aligned}
$$

$e\_list := [\emptyset];$

$test := [\,];$

for $i := 1$ to $|\mathcal{X}|$ do $counts[i] := 0;$

for $position := 1$ to $test\_lenght$ do

    $new\_e\_list := [\,];$

    for all $\mathbf{e} \in e\_list$ do

        $i := most\_informative\_X(\mathbf{e});$

        $counts[i] := counts[i] + P(\mathbf{e});$

        for all $x_i \in \mathbb{X}_i$ do

            $append(new\_e\_list, \{\mathbf{e} \cup \{X_i = x_i\}\});$

    $e\_list := new\_e\_list;$

    $i^\star := \arg\max_i \; counts[i];$

    $append(test, X_{i^\star});$

    $counts[i^\star] := 0;$

$return(test);$

CAT for basic operations with fractions

Examples of tasks:

$$T_1\text{:}\quad \left(\frac{3}{4}\cdot\frac{5}{6}\right)-\frac{1}{8} \quad = \quad \frac{15}{24}-\frac{1}{8}=\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

$$T_2\text{:}\quad \frac{1}{6}+\frac{1}{12} \quad = \quad \frac{2}{12}+\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$$

$$T_3\text{:}\quad \frac{1}{4}\cdot 1\frac{1}{2} \quad = \quad \frac{1}{4}\cdot\frac{3}{2}=\frac{3}{8}$$

$$T_4\text{:}\quad \left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot\left(\frac{1}{3}+\frac{1}{3}\right) \quad = \quad \frac{1}{4}\cdot\frac{2}{3}=\frac{2}{12}=\frac{1}{6}\ .$$

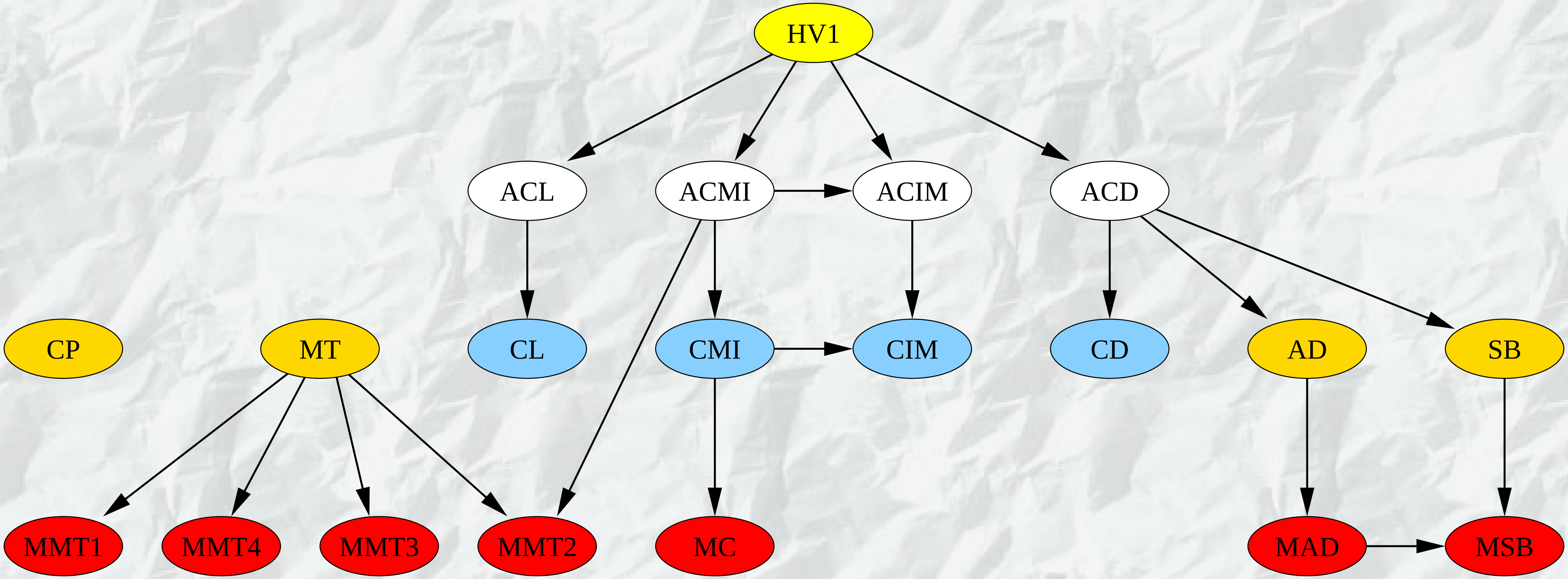
Elementary and operational skills

CP	Comparison (common numerator or denominator)	$\frac{1}{2} > \frac{1}{3}, \quad \frac{2}{3} > \frac{1}{3}$
AD	Addition (comm. denom.)	$\frac{1}{7} + \frac{2}{7} = \frac{1+2}{7} = \frac{3}{7}$
SB	Subtract. (comm. denom.)	$\frac{2}{5} - \frac{1}{5} = \frac{2-1}{5} = \frac{1}{5}$
MT	Multiplication	$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$
CD	Common denominator	$\left(\frac{1}{2}, \frac{2}{3}\right) = \left(\frac{3}{6}, \frac{4}{6}\right)$
CL	Cancelling out	$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}$
CIM	Conv. to mixed numbers	$\frac{7}{2} = \frac{3 \cdot 2+1}{2} = 3\frac{1}{2}$
CMI	Conv. to improp. fractions	$3\frac{1}{2} = \frac{3 \cdot 2+1}{2} = \frac{7}{2}$

Misconceptions

Label	Description	Occurrence
MAD	$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$	14.8%
MSB	$\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$	9.4%
MMT1	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a \cdot c}{b}$	14.1%
MMT2	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a+c}{b \cdot b}$	8.1%
MMT3	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$	15.4%
MMT4	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b+d}$	8.1%
MC	$a\frac{b}{c} = \frac{a \cdot b}{c}$	4.0%

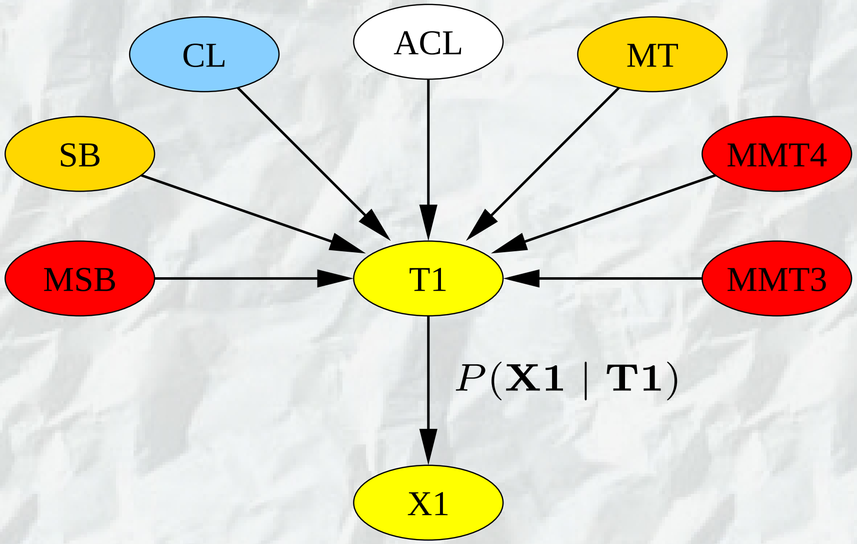
Student model



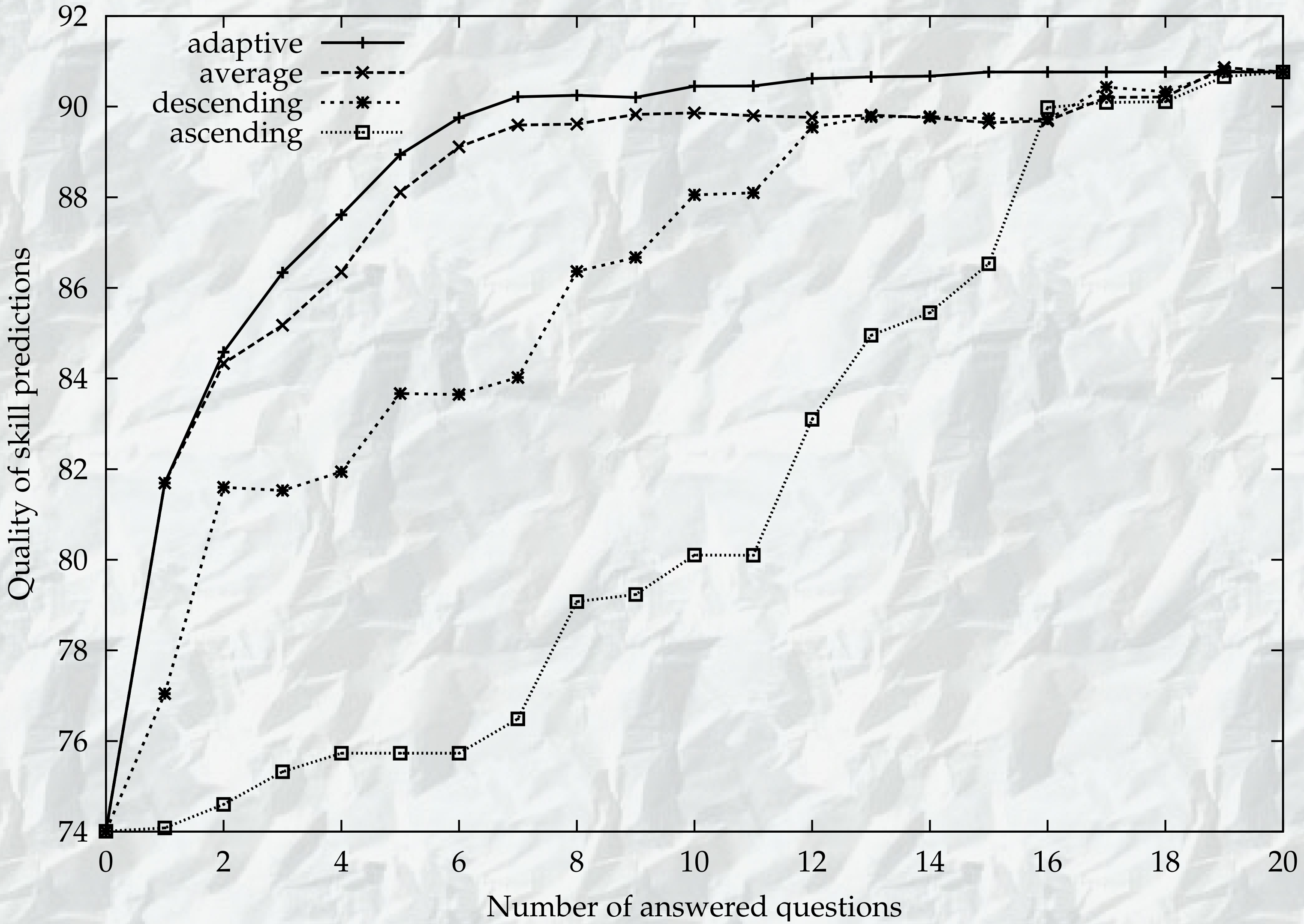
Evidence model for task $T1$

$$\left(\frac{3}{4}\cdot\frac{5}{6}\right)-\frac{1}{8}=\frac{15}{24}-\frac{1}{8}=\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

$$T1 \quad \Leftrightarrow \quad MT \;\&\; CL \;\&\; ACL \;\&\; SB \;\&\; \neg MMT3 \;\&\; \neg MMT4 \;\&\; \neg MSB$$



Skill Prediction Quality



Conclusions

- Empirical evidence shows that educational testing can benefit from application of Bayesian networks. Adaptive tests may substantially **reduce the number of questions** that are necessary to be asked.
- **Method for** the design of a **fixed test** provided good results on tested data. It may be regarded as a good cheap alternative to computerized adaptive tests when they are not suitable.
- One theoretical problem related to application of Bayesian networks to educational testing is **efficient inference exploiting deterministic relations** in the model. This problem was topic of our UAI 2002 paper.