Integrating inconsistent data in a probabilistic model

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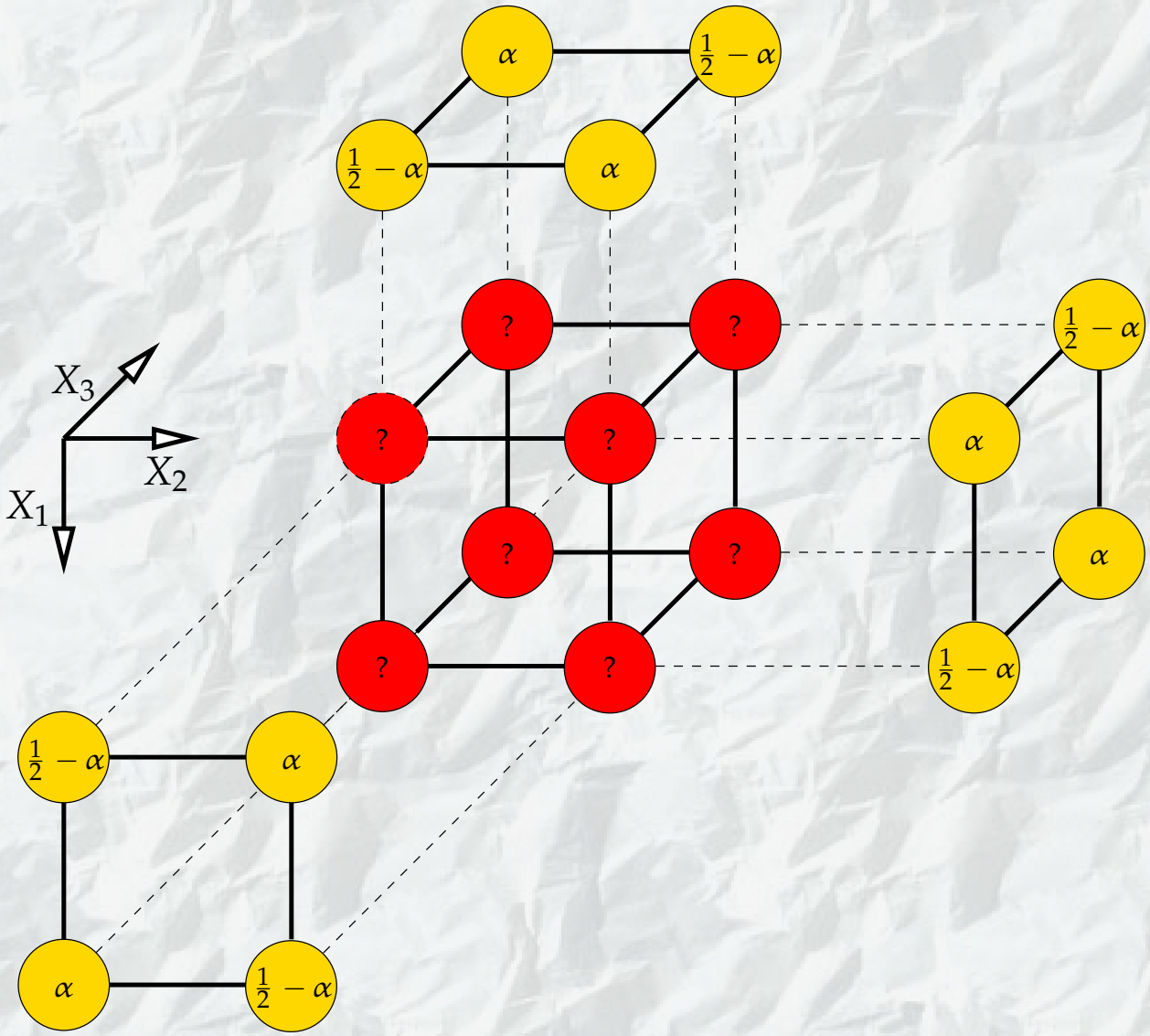
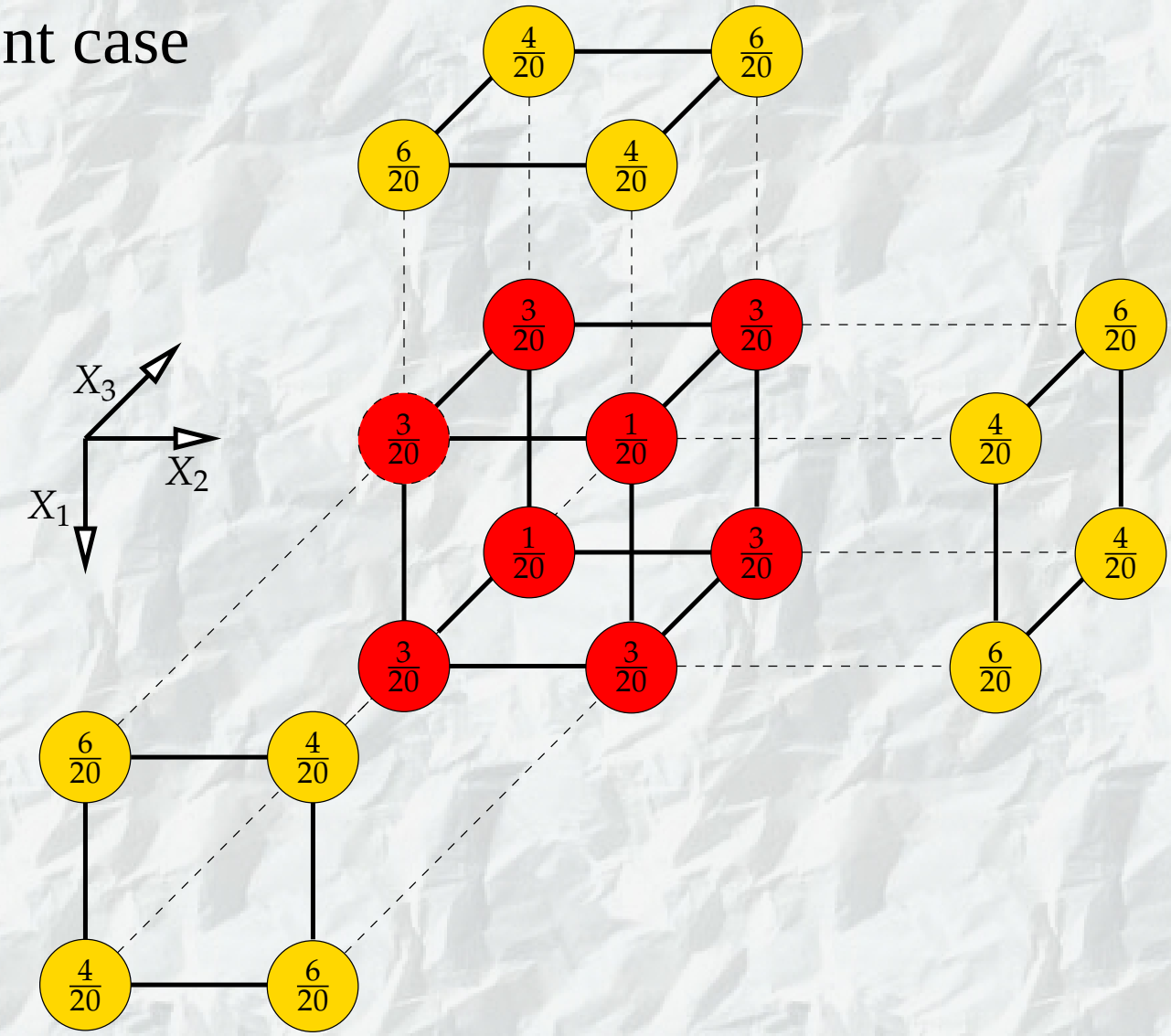
Knowledge integration

- Discrete random variables X_i indexed by natural numbers from $V = \{1, \ldots, n\} \subset \mathbb{N}$.
- Low-dimensional probability distributions P_j, $j = 1, \ldots, k$ defined on variables $\{X_\ell\}_{\ell \in E_j}$, $E_j \subseteq V$.
- *Knowledge integration* is the process of building a joint probability distribution $Q(X_1, \ldots, X_n)$ from a set of low-dimensional probability distributions $\mathcal{P} = \{P_1, \ldots, P_k\}$.


X_3
X_2
X_1
α
$\frac{1}{2}-\alpha$
?

Consistent case

$$\alpha = \frac{4}{20}$$

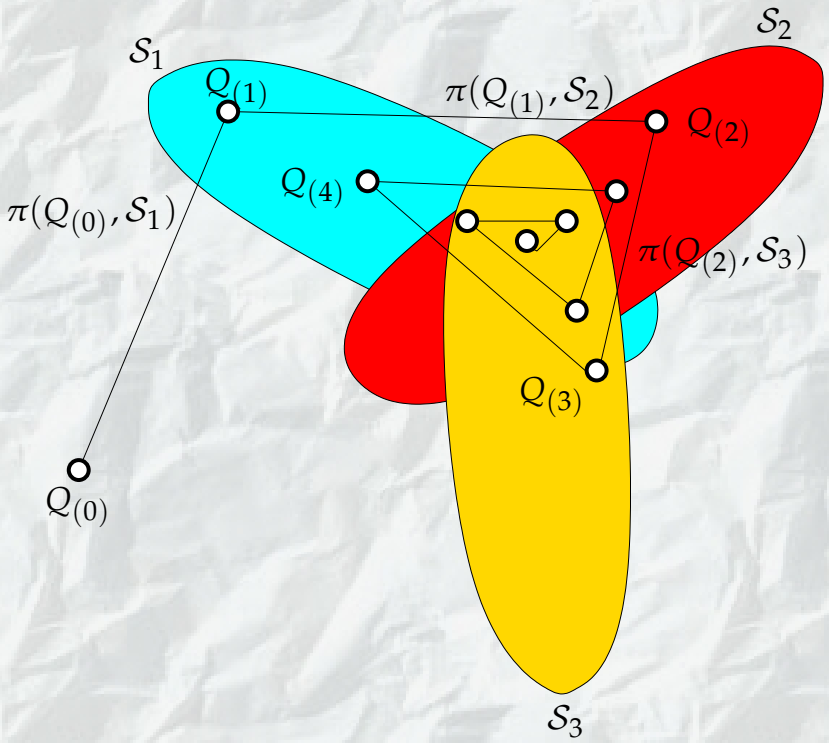


- input set $\mathcal{P} = \{P_1, \ldots, P_k\}$
- set of all distributions having P_j as its marginal
  $\mathcal{S}_j = \{Q : Q^{E_j} = P_j\}$
- set of all distributions having $\{P_1, \ldots, P_k\}$ as its marginals
  $\mathcal{S} = \cap_{j=1}^{k} \mathcal{S}_j$
- I-projection of Q_0 to $\mathcal{S}$
  $\pi(Q_0, \mathcal{S}) = \arg\min_{Q \in \mathcal{S}} I(Q \parallel Q_0)$
- Kullback-Leibler divergence

$$I(P \parallel Q) \quad = \quad \sum_x P(X = x) \log \frac{P(X = x)}{Q(X = x)}$$

Iterative Proportional Fitting Procedure (IPFP)

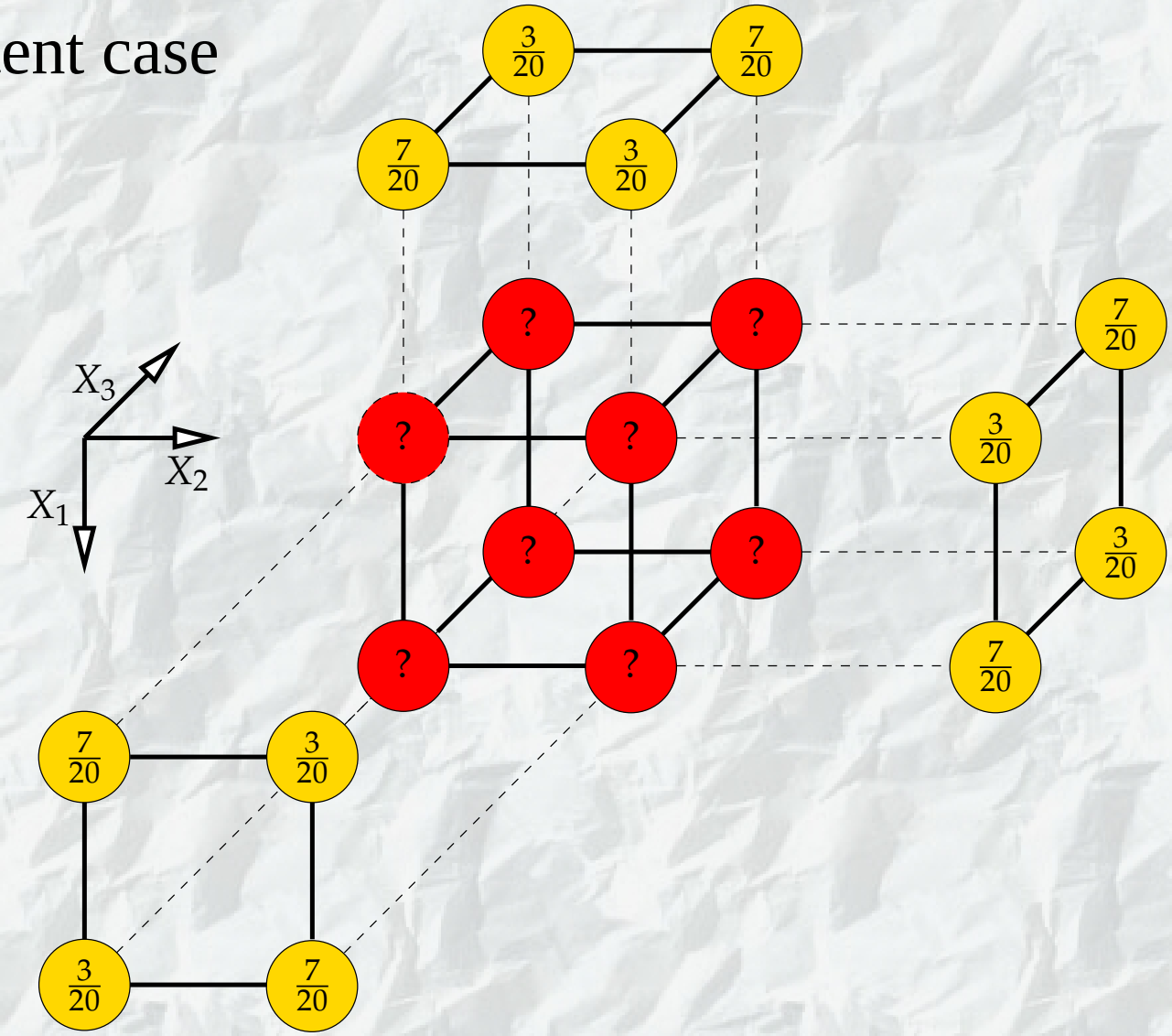
Deming & Stephan, 1940



$$\pi(Q, \mathcal{S}_j) \quad = \quad Q \frac{P_j}{Q^{E_j}}$$

Inconsistent case

$\alpha = \frac{3}{20}$



Inconsistent input set $\mathcal{P} = \{P_1, \ldots, P_k\}$

It means that $\mathcal{S} = \cap_{j=1}^{k} \mathcal{S}_j = \emptyset$.

$Q(X_1, \ldots, X_n)$ is required to:

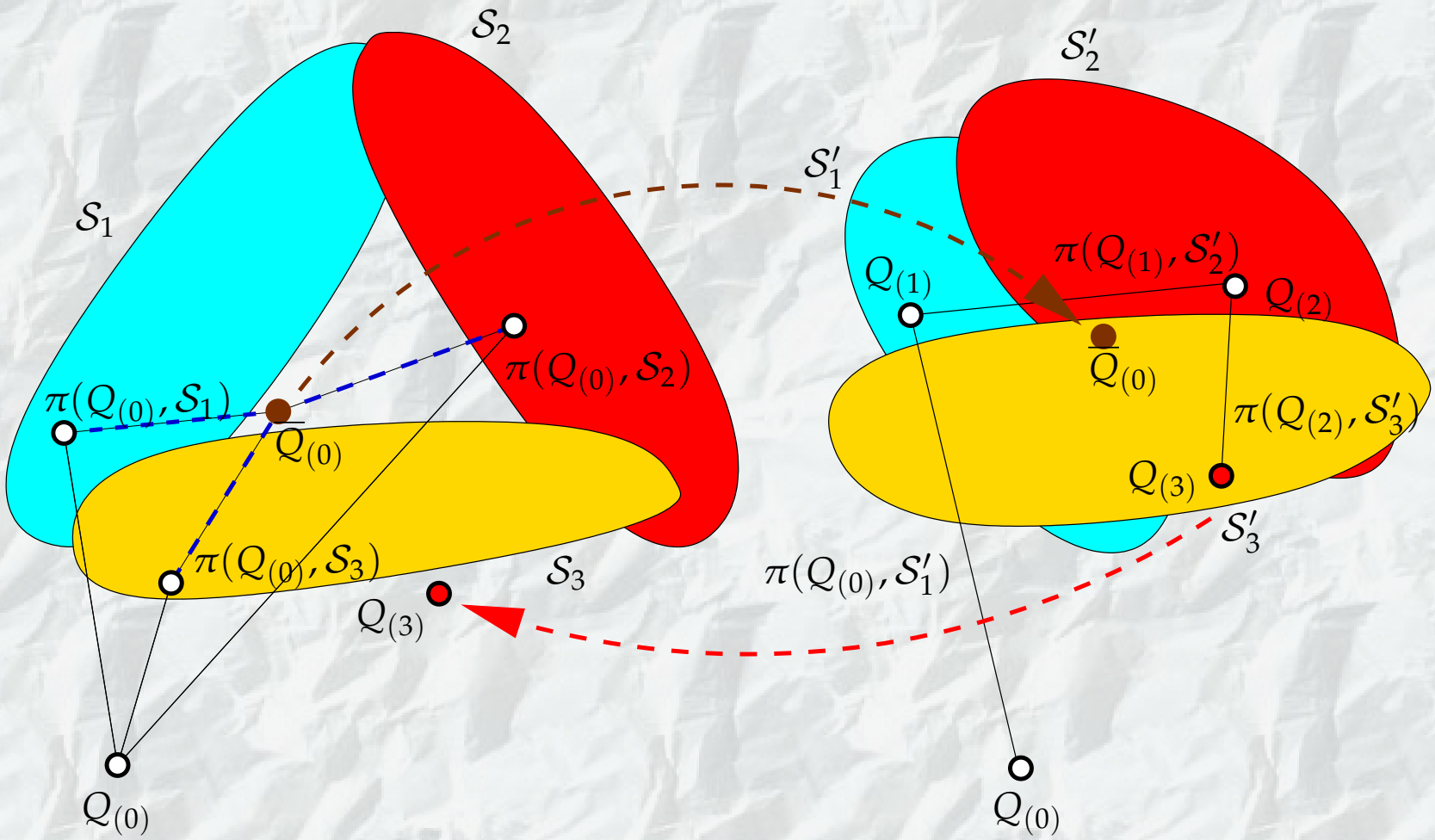
- **minimize I-aggregate** with respect to $\mathcal{P}$:

$$\sum_{P_j \in \mathcal{P}} w_j \cdot I(P_j \parallel Q^{E_j})$$

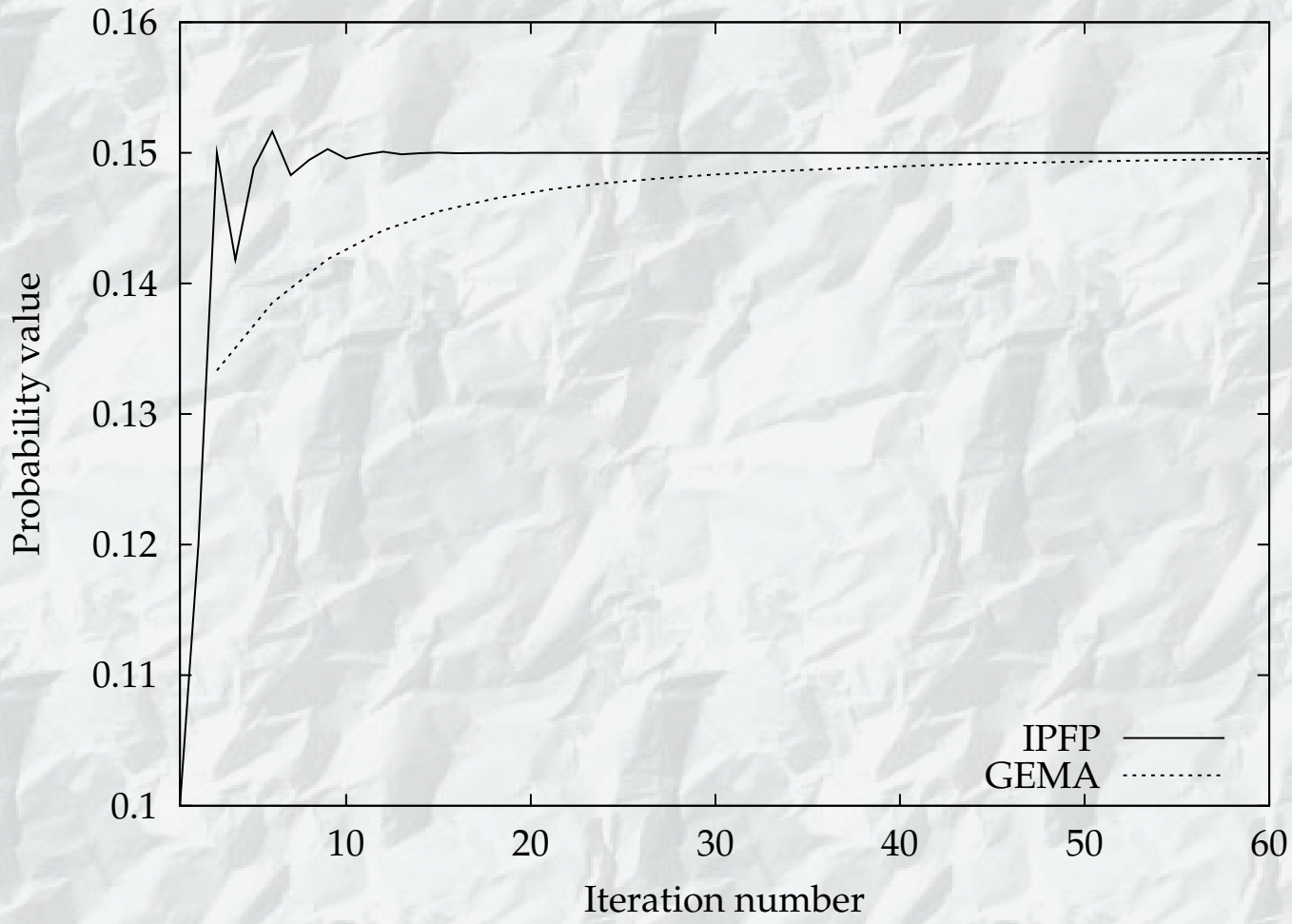
- **factorize** with respect to $\mathcal{E} = \{E_1, \ldots, E_k\}$:
  there exist potentials $\psi_{E_i} : \mathbb{X}^{E_i} \mapsto \mathbb{R}, i = 1, 2, \ldots, k$ such that for all $x \in \mathbb{X}$

$$Q(x) \quad = \quad \prod_{E_i \in \mathcal{E}} \psi_{E_i}(x^{E_i}) \ .$$

GEMA


$\mathcal{S}_2$
$\mathcal{S}_1$
$\pi(Q_{(0)}, \mathcal{S}_1)$
$\overline{Q}_{(0)}$
$\pi(Q_{(0)}, \mathcal{S}_2)$
$\pi(Q_{(0)}, \mathcal{S}_3)$
$\mathcal{S}_3$
$Q_{(3)}$
$Q_{(0)}$
$\mathcal{S}'_1$
$\mathcal{S}'_2$
$Q_{(1)}$
$\pi(Q_{(1)}, \mathcal{S}'_2)$
$Q_{(2)}$
$\overline{Q}_{(0)}$
$\pi(Q_{(2)}, \mathcal{S}'_3)$
$Q_{(3)}$
$\mathcal{S}'_3$
$\pi(Q_{(0)}, \mathcal{S}'_1)$
$Q_{(0)}$

IPFP and GEMA on the consistent input set



IPFP and GEMA on the inconsistent input set