What are imsets and what they are good for

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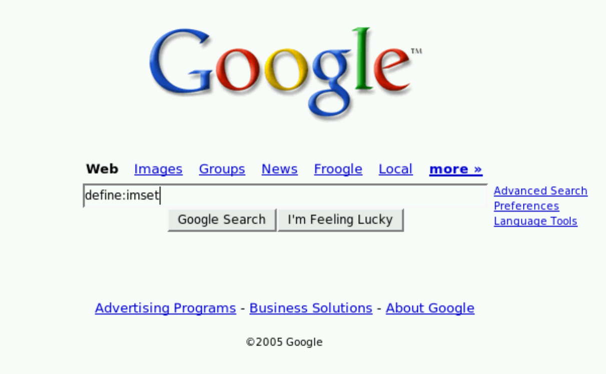
ÚTIA AV ČR

VŘSR, 11. - 13. 11. 2005

What is an imset?

Ask Google



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In Egyptian mythology Imset was a funerary deity, one of the Four sons of Horus, who were associated with the canopic jars, specifically the one which contained the liver.

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What is an imset? (formal definition)

- N ... a finite set
- $\mathcal{P}(N)$ ... power set of N
- $\mathbb{Z}$ ... set of all integers

Definition

Imset u is a function $u : \mathcal{P}(N) \mapsto \mathbb{Z}$.

Function $m : \mathcal{P}(N) \mapsto \mathbb{N}$ is sometimes called multiset. Thus, imset is an abbreviation from **I**nteger valued **M**ulti**SET**. Studený (2001)

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What is an imset? (an example)

Let $N = \{a, b, c\}$. An imset u over N is

$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{b,c\}$	$\{a,c\}$	$\{a,b,c\}$
0	0	+1	0	-1	-1	0	+1

A convention:

$$\delta_A(B) = \begin{cases} 1 & \text{if } A = B \\ 0 & \text{otherwise} \end{cases}$$

$$\forall B \subseteq N: \; u(B) = \sum_{A \subseteq N} u(A) \cdot \delta_A(B)$$

$$u = \sum_{A \subseteq N} c_A \cdot \delta_A$$

Using the convention we will write

$$u = \delta_{\{b\}} - \delta_{\{a,b\}} - \delta_{\{b,c\}} + \delta_{\{a,b,c\}}$$

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Elementary and structural imsets

Definition (Elementary imset)

Let $K \subseteq N$, $a, b \in N \setminus K$, and $a \neq b$.
Elementary imset is defined by the formula

$$u_{\langle a,b|K\rangle} \quad = \quad \delta_{\{a,b\}\cup K} + \delta_K - \delta_{\{a\}\cup K} - \delta_{\{b\}\cup K}$$

Let $\mathcal{E}(N)$ denote the set of all elementary imsets.

Definition (Structural imset)

An imset u is structural iff

$$n \cdot u \quad = \quad \sum_{v \in \mathcal{E}(N)} k_v \cdot v \quad ,$$

where $n \in \mathbb{N}$ and $v \in \mathcal{E}(N) : k_v \in \mathbb{N} \cup \{0\}$

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Conditional Independence (CI) Models (definition)

Definition (Disjoint triplet over N)

Let $A, B, C \subseteq N$ be pairwise disjoint. Then $\langle A, B \mid C \rangle$ denotes a disjoint triplet over N. $\mathcal{T}(N)$ will denote the class of all possible disjoint triplets over N.

Definition (CI-statement)

Let $\langle A, B \mid C \rangle$ be a disjoint triplet over N. Then "A is conditionally independent of B given C" is an CI-statement, written as $A \perp\!\!\!\perp B \mid C$.

Definition (CI-model)

CI-model is a set of CI-statements.

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CI-model is a set of CI-statements.

CI-model (an example by F. V. Jensen)

Assume three variables:

- h is the length of hair *long*, *short*,
- s is stature that has states $< 168cm$, $> 168cm$,
- g is gender that takes states *male*, *female*.

If we do not know the gender of a person

- seeing the length of his/her hair will tell us more about the gender, i.e. $not(h \perp\!\!\!\perp g)$,
- this will focus our belief on his/her stature, i.e. $not(g \perp\!\!\!\perp s)$ and $not(h \perp\!\!\!\perp s)$.

But if we know the gender of a person then length of hair gives us no extra clue on his stature, i.e., $h \perp\!\!\!\perp s \mid g$.
Thus, we get a CI-model consisting of just one CI-statement $h \perp\!\!\!\perp s \mid g$.

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CI-objects

Definition (CI-object)

CI-object over N is a mathematical object defined over N that can be used to generate a CI-model.

Classes of CI-objects

1. Probability distributions (PDs) from a given class defined over random variables from N. E.g., discrete PDs over N.
2. Undirected graphs (UGs) over the set of nodes N.
3. Acyclic directed graph (DAGs) over the set of nodes N.
4. Structural imsets over N.

CI-objects generating the CI-model $a \perp\!\!\!\perp c \mid b$

1. $\forall v_a, v_b, v_c$:
   $P(a = v_a, c = v_c \mid b = v_b) \; = \; P(a = v_a \mid b = v_b) \cdot P(c = v_c \mid b = v_b)$
2. $a \; - \; b \; - \; c$
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How CI-objects generate CI-models?

In UGs

Definition (Separation criteria)

$A \perp\!\!\!\perp B \mid C$ is represented in UG G if every route in G between a node in A and a node in B contains a node from C.

In imsets

Definition

$A \perp\!\!\!\perp B \mid C$ is represented in an imset u if there exists $k \in \mathbb{N}$ such that

$$k \cdot u = u_{\langle A,B|C\rangle} + w$$

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Perfectly Markovian probability distribution

- $M(P)$ ... a CI-model generated by a discrete probability distribution P over N
- $\mathcal{M}$ ... the class of all CI-models generated by a discrete probability distribution over N
- $\mathcal{O}$ ... a considered class of CI-objects (e.g., DAGs, UGs, structural imsets)

Definition (Perfectly Markovian)

A probability distribution P is perfectly Markovian with respect to an object $O \in \mathcal{O}$ if for every $\langle A, B \mid C \rangle \in \mathcal{T}(N)$

$$A \perp\!\!\!\perp B \mid C \text{ is represented in } P \iff A \perp\!\!\!\perp B \mid C \text{ is represented in } M \ .$$

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- $\mathcal{M}$ ... the class of all CI-models generated by a discrete probability distribution over N
- $\mathcal{O}$ ... a considered class of CI-objects (e.g., DAGs, UGs, structural imsets)

Definition (Perfectly Markovian)

A probability distribution P is perfectly Markovian with respect to an object $O \in \mathcal{O}$ if for every $\langle A, B \mid C \rangle \in \mathcal{T}(N)$

$$A \perp\!\!\!\perp B \mid C \text{ is represented in } P \iff A \perp\!\!\!\perp B \mid C \text{ is represented in } M \; .$$

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Properties of classes of CI-objects (definitions)

Definition (Faithfulness of $\mathcal{O}$)

For every CI-object $O \in \mathcal{O}$ there exists a CI-model $M(P)$ from $\mathcal{M}$ such that P is perfectly Markovian with respect to O.

Definition (Completeness of $\mathcal{O}$)

For every CI-model $M(P)$ from $\mathcal{M}$ there exists a CI-object $O \in \mathcal{O}$ such that P is perfectly Markovian with respect to O.

Definition (Uniqueness of $\mathcal{M}$)

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	Faithfulness	Completeness	Uniqueness
UGs	yes	no	yes
DAGs	yes	no	no
structural imsets	no	yes	no

Number of CI-models generated by CI-objects

$\|N\|$	UGs	DAGs	$\|\mathcal{M}\|$
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4	64	185	18478

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- In contrast to graphical probabilistic models (UGs and DAGs) the class of structural imsets is complete, i.e. it can describe all CI-models generated by a discrete probability distribution.
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