Thoughts
on belief and model revision
with uncertain evidence

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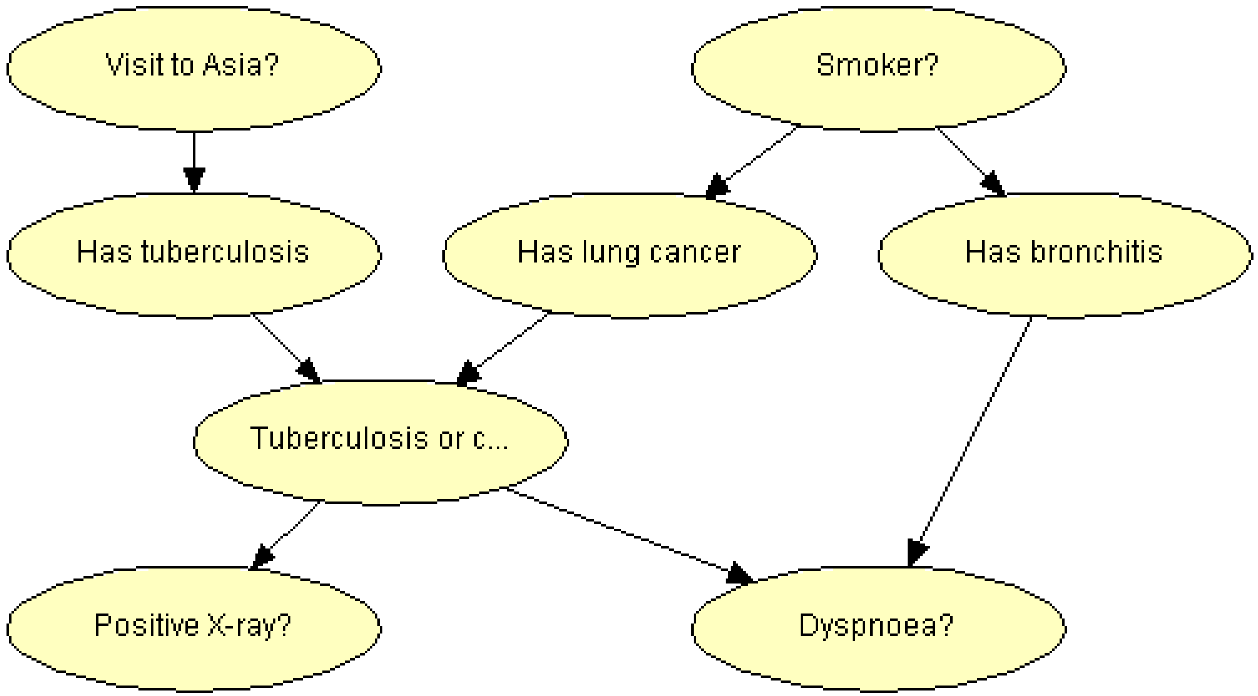
Contents:

- What is a Bayesian network? A simple example - “Chest clinic” .
- Belief revision with new evidence (certain and uncertain)
- Reliability of information sources
- Belief revision based on sources’ reliability
- Belief revision in a complex probabilistic model
- Belief vs. Model revision
- Revision of a model parameter
- Summary

Chest Clinic (Lauritzen and Spiegelhalter, 1988)
a Bayesian network example



Visit to Asia?

yes	0.01
no	0.99

Smoker?

yes	0.5
no	0.5

Has tuberculosis

Visit to Asi...	yes	no
yes	0.05	0.01
no	0.95	0.99

Has lung cancer

Smoker?	yes	no
yes	0.1	0.01
no	0.9	0.99

Has bronchitis

Smoker?	yes	no
yes	0.6	0.3
no	0.4	0.7

Positive X-ray?

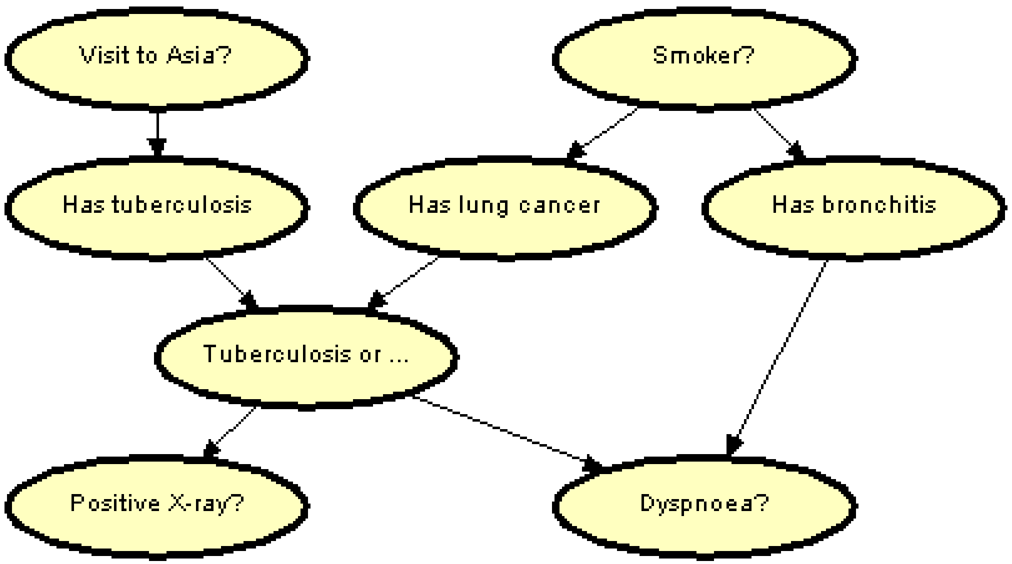
Tuberculo...	yes	no
yes	0.98	0.05
no	0.02	0.95

Tuberculosis or cancer

Has tuberc...	yes		no	
Has lung c...	yes	no	yes	no
yes	1	1	1	0
no	0	0	0	1

Dyspnoea?

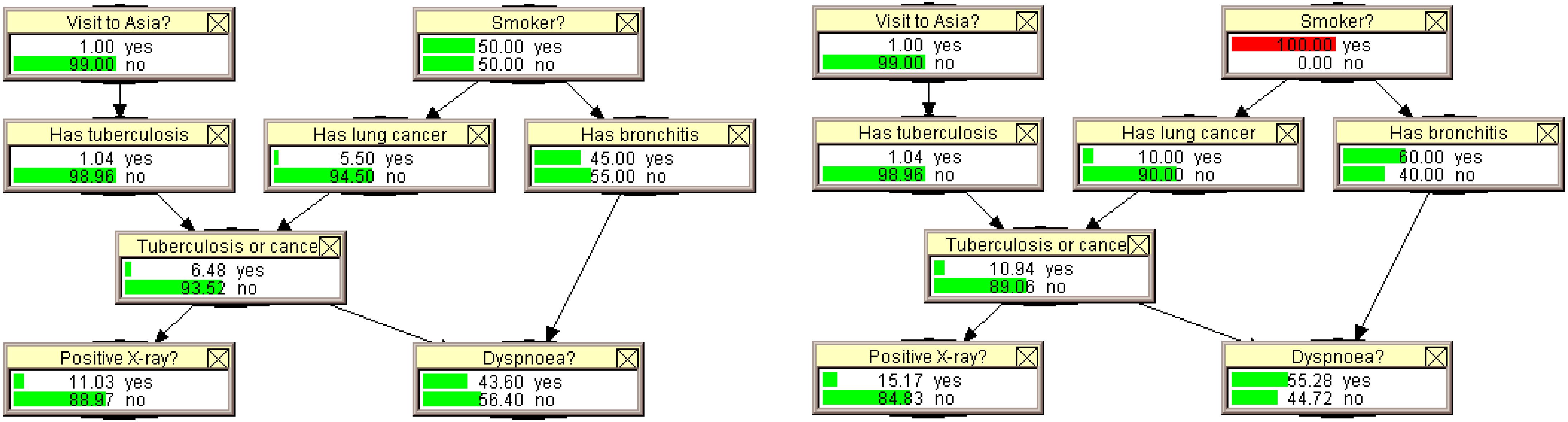
Has bronc...	yes		no	
Tuberculo...	yes	no	yes	no
yes	0.9	0.8	0.7	0.1
no	0.1	0.2	0.3	0.9



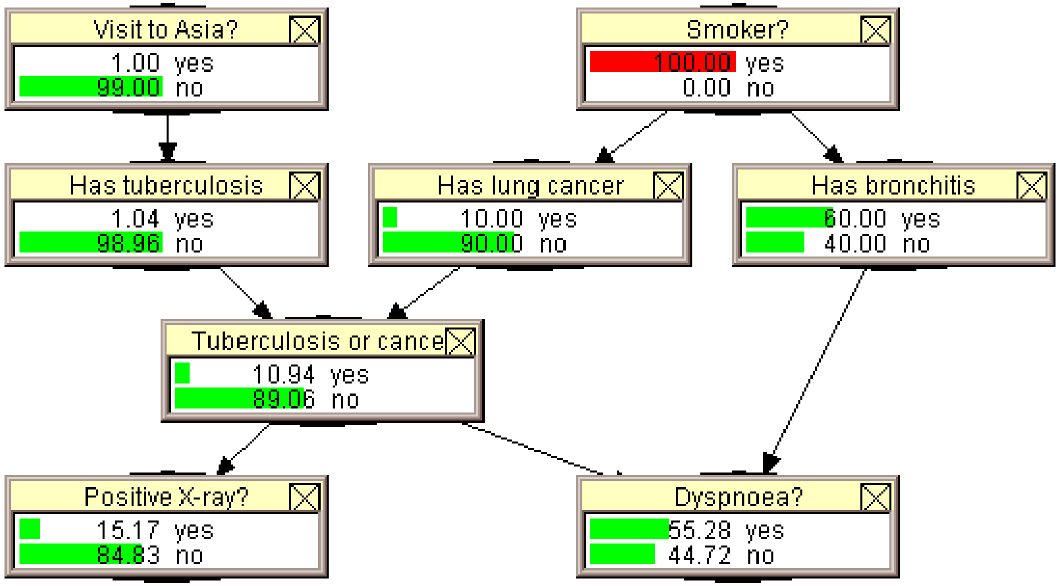
Belief revision with new evidence using Bayes' rule

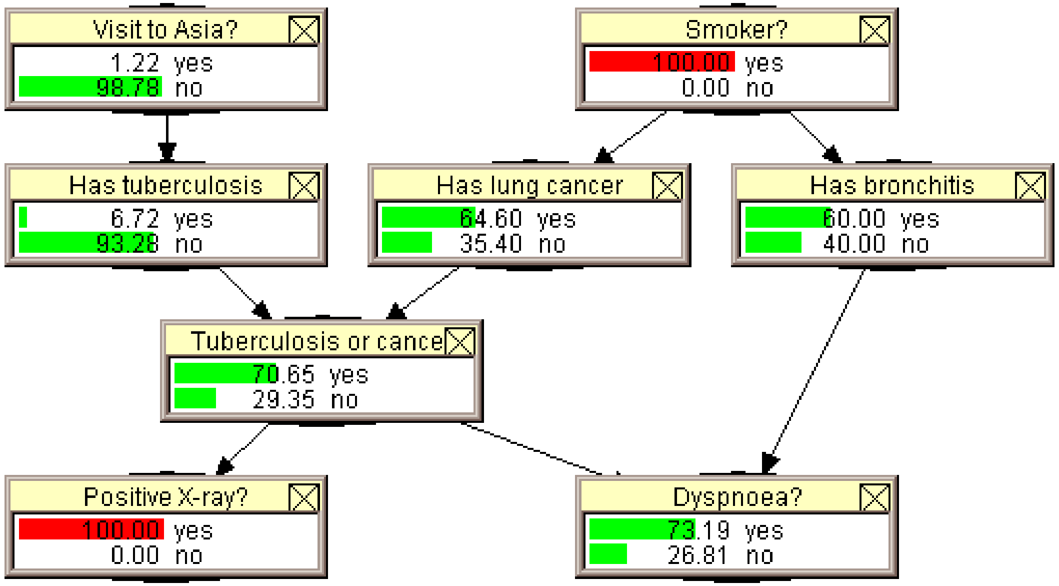
$$P(Bronchitis = yes \mid Smoker = yes) =$$

$$= \frac{\text{number of smokers with bronchitis}}{\text{number of smokers}} = \frac{P(Bronchitis = yes, Smoker = yes)}{P(Smoker = yes)}$$

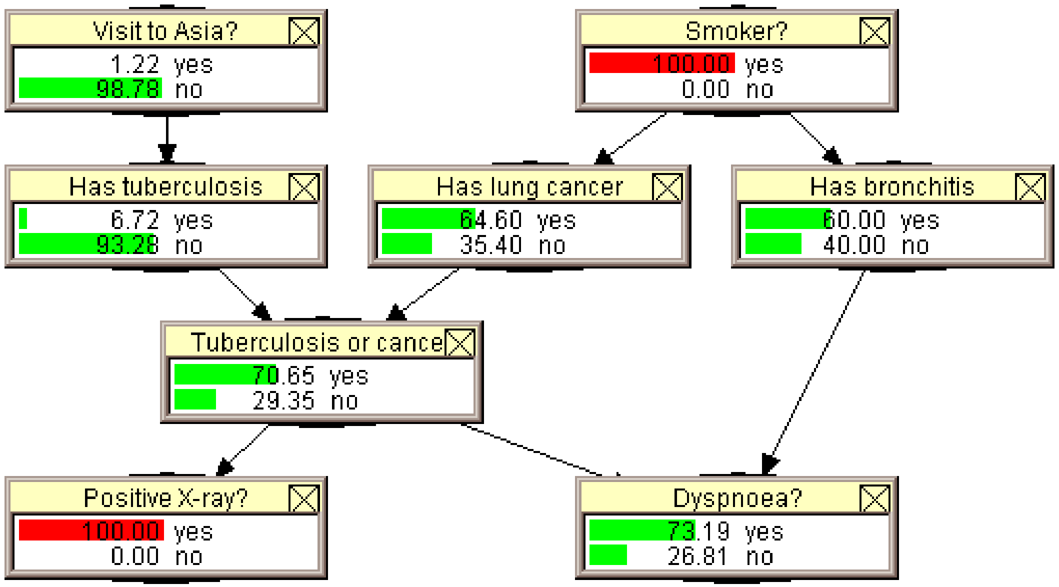


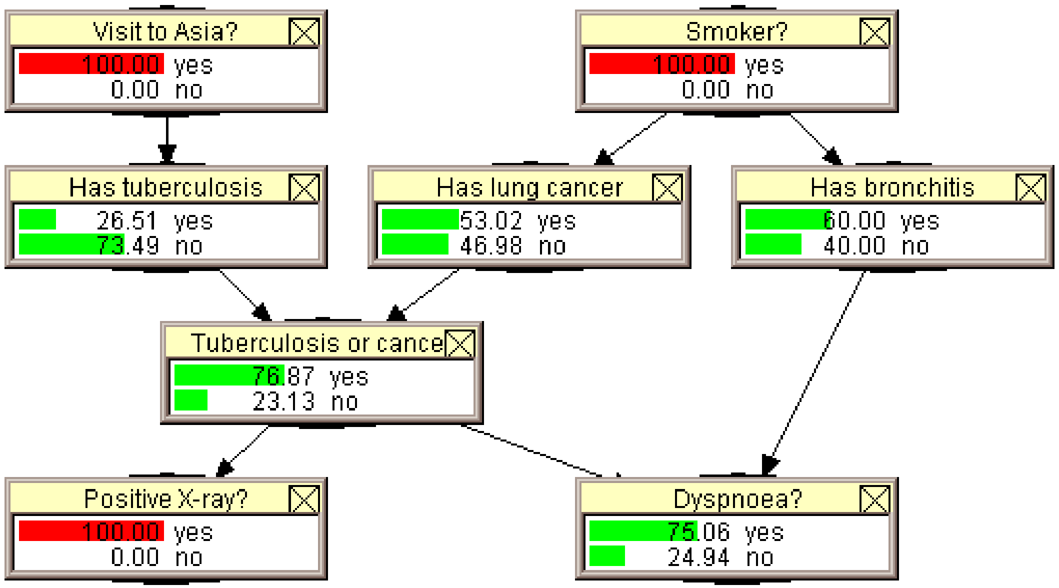
Belief revision with new evidence - propagation upwards





Belief revision with new evidence - explaining away





The problem with uncertain evidence

- How should we revise our beliefs with uncertain evidence?

Examples

- "Based on what the physician sees on an x-ray she believes it is highly probable that the patient has cancer."
- "The color of this cloth seems red, but I'm not sure, since it is quite dark and it is far away."
- "The alarm in my house went on. There is a chance of burglary, but I know that reliability of the sensors is only 60%."
- "A intelligence agency got report from an agent. They know that this source is usually very reliable."

Our goal is to use the *standard probability framework* (with frequentist or subjective belief interpretation) to deal with uncertain evidence (in cases where it is suitable).

Reliability of information sources

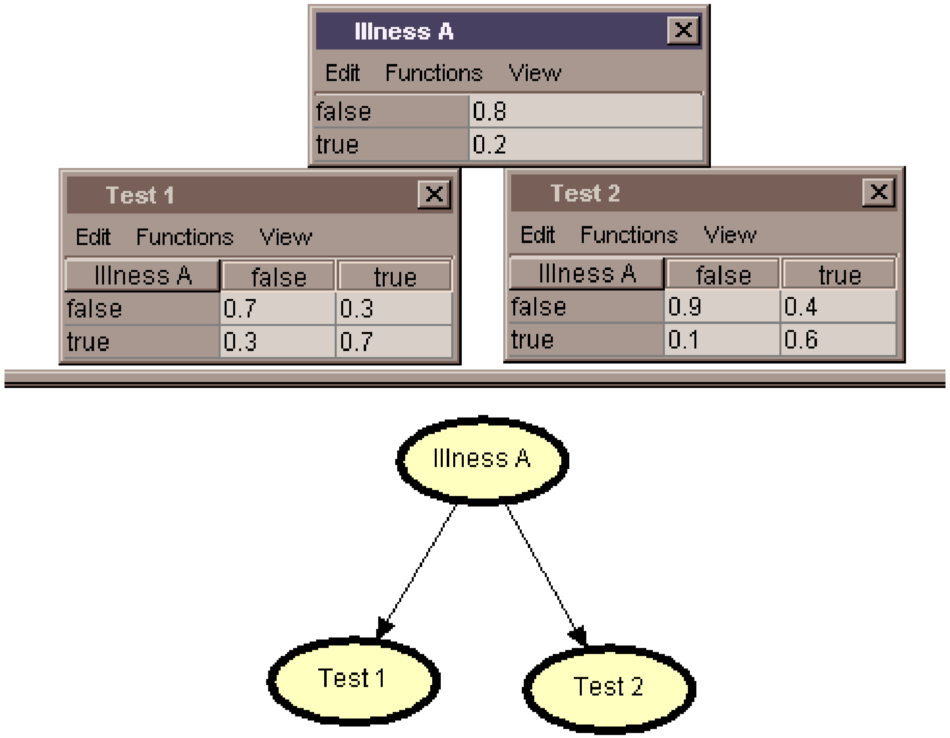
A ... a variable of our interest,

T ... a test, report, observation.

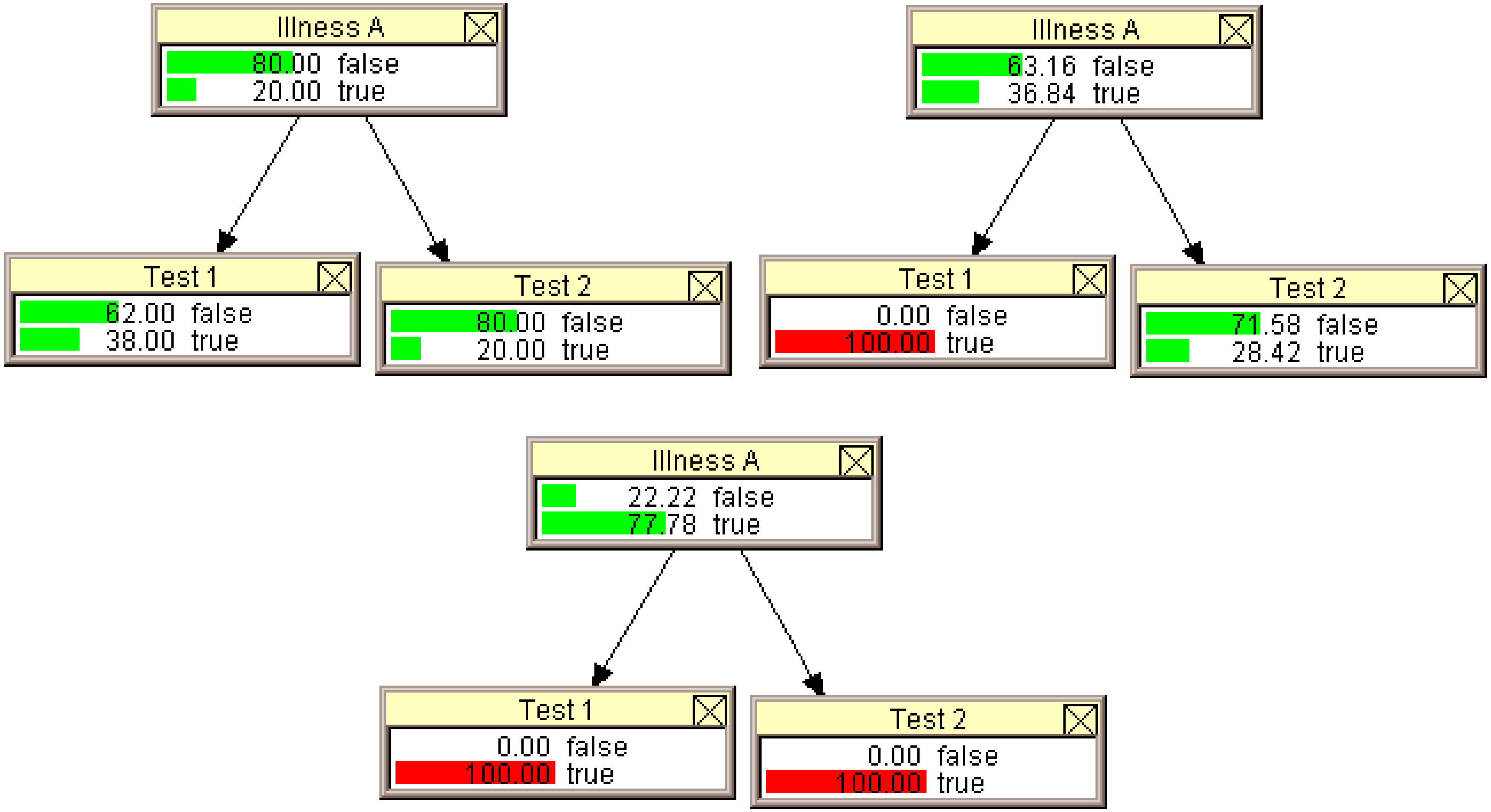
accuracy	$P(A = T) = \frac{tp+tn}{tp+tn+fp+fn}$
positive predictive value (precision)	$P(A = yes \mid T = yes) = \frac{tp}{tp+fp}$
negative predictive value	$P(A = no \mid T = no) = \frac{tn}{fn+tn}$
true positive rate (recall or sensitivity)	$P(T = yes \mid A = yes) = \frac{tp}{tp+fn}$
true negative rate (specificity)	$P(T = no \mid A = no) = \frac{tn}{fp+tn}$
false positive rate	$P(T = yes \mid A = no) = \frac{fp}{fp+tn}$
false negative rate	$P(T = no \mid A = yes) = \frac{fn}{tp+fn}$

Belief revision based on sources’ reliability

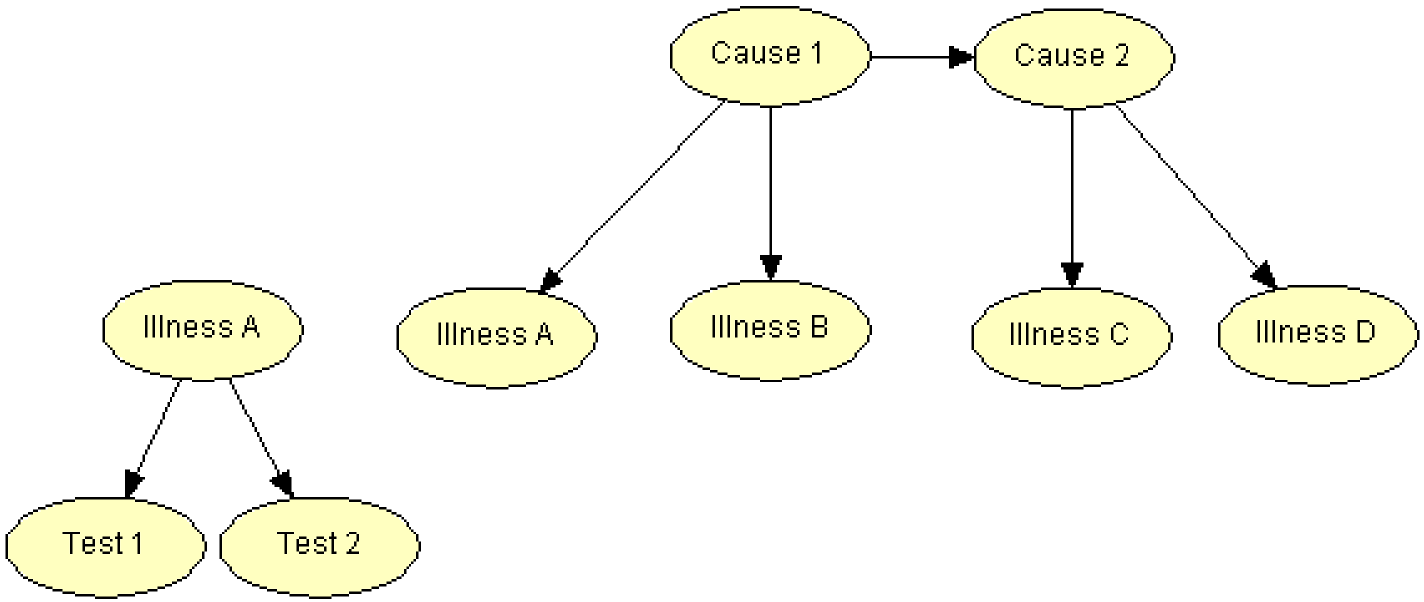
- sensitivity of T, i.e. $P(T = yes \mid A = yes)$
- specificity of T, i.e. $P(T = no \mid A = no)$, and
- an observed result $T = t$



Example of two tests



Belief revision in a complex probabilistic model $P(V)$



Expert calculations:

$$
\begin{aligned}
P'(A) &= P(A \mid T_1, \ldots, T_n) \\
&= c \cdot P(A) \cdot P(T_1 \mid A) \cdot P(T_2 \mid A)
\end{aligned}
$$

Belief revision in the model:

$$P'(V) = P(V \setminus A \mid A) \cdot P'(A)$$

Belief vs. Model revision

a fundamental difference

- The posterior probabilities after *belief revision* inform about the properties of a tested individual, a performed event, etc.
- The posterior probabilities after a *model revision* still correspond to the properties of the tested population of individuals, events, etc.

Model revision

revision of a model parameter

- for each parameter only one value
  $\longrightarrow$ *maximum likelihood estimation*

- posterior probabilities defined over the model parameters
  $\longrightarrow$ *Bayesian statistics*

Example - maximum likelihood estimate of a parameter

Two variables (their states are known in the tested population):

- test result T (positive/negative)
- illness A (sick/non-sick).

Likelihood of observed data D for a given model $P(T, A)$

$$L(P \mid D) \quad = \quad \prod_{(t,a)} P(T = t, A = a)^{n(t,a)}$$

Sensitivity r of test T was defined as $P(T = yes \mid A = yes)$.
Maximum likelihood estimate:

$$\hat{r} \quad = \quad \arg\max_{r} L(P \mid D)$$

Example - combining two maximum likelihood estimates

Two experts evaluated sensitivity r of a test. They computed their maximum likelihood estimate

$$\hat{r}_i = \frac{n_i(T = yes, A = yes)}{n_i(A = yes)} \text{ for } i = 1, 2.$$

$$L(P \mid D) = \prod_{(t,a)} P(T = t, A = a)^{n_1(t,a)} \cdot P(T = t, A = a)^{n_2(t,a)}$$

Maximum likelihood estimate of r is weighted average of $\hat{r}_1$ and $\hat{r}_2$

$$\begin{aligned}\hat{r} &= \frac{n_1(A = yes)}{n_1(A = yes) + n_2(A = yes)} \cdot \hat{r}_1 + \frac{n_2(A = yes)}{n_1(A = yes) + n_2(A = yes)} \cdot \hat{r}_2 \\ &= w_1(yes) \cdot \hat{r}_1 + (1 - w_1(yes)) \cdot \hat{r}_2\end{aligned}$$

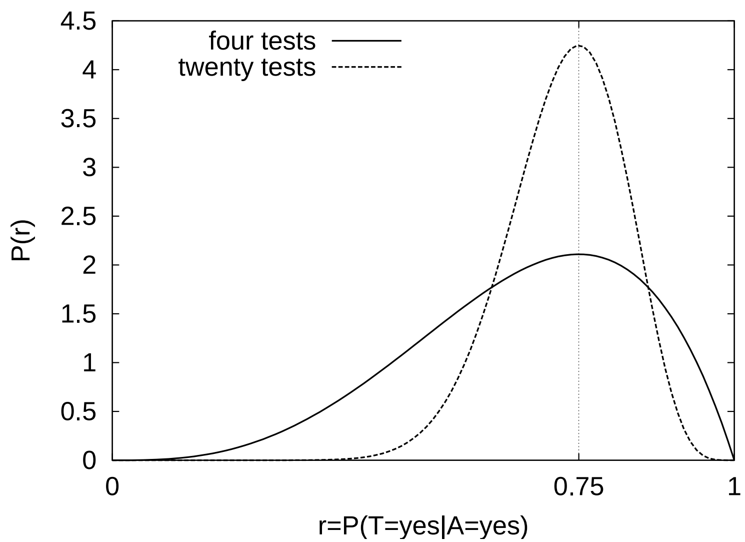
Example - posterior probability of model parameters

Instead of a single value $\hat{r}$ we take whole distribution.

(1) $n_1(A = yes) = n_2(A = yes) = 2$

(2) $n_1(A = yes) = n_2(A = yes) = 10$

In both cases $\hat{r}_1 = 0.7$ and $\hat{r}_2 = 0.8$ and the prior distribution is uniform.



Summary

- Bayesian networks are suitable for reasoning with uncertainty.
- Standard efficient methods for belief revision with certain evidence are available (since 90's).
- Different methods for belief revision with uncertain evidence can be used depending on what is the reported belief referring to (e.g. sensitivity of a test or a final expert belief in the presence of an illness)
- There is a fundamental difference between belief and model revision.
- Sometimes most likely values of model parameters are sufficient while in some situation it is more appropriate to use whole posterior probability distribution of parameter values.