Noisy logical connectives in Bayesian networks

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Outline

- Brief introduction to Causal Probabilistic (CP) logic

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- Converting a CP-theory to a Bayesian network

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- Converting a CP-theory to a Bayesian network
- Efficient probabilistic inference with Bayesian networks

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If *John* and *Mary* both buy dinner, it is possible that they both buy *spaghetti*. If they buy something different, they can choose what they will have for dinner, because two meals have been bought.

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If p is a predicate symbol of arity n and $a_1, \ldots, a_n$ are ground terms then $p(a_1, \ldots, a_n)$ is a ground atom. E.g., $shops(paul)$, $bought(spaghetti)$.

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Herbrand interpretation is a subset of the Herbrand base that is true.

Causal Probabilistic (CP) Logic (Vennekens, 2007)

Definition

A rule is the statement

$$(p_1 : \alpha_1) \vee \ldots \vee (p_n : \alpha_n) \quad \leftarrow \quad \varphi \ ,$$

where φ is a conjuction of ground atoms or their negations, the p_i are ground atoms and the α_i are non-zero probabilities with $\sum \alpha_i \leq 1$.

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Example

$$(bought(spaghetti) : 0.3) \vee (bought(fish) : 0.7) \quad \leftarrow \quad shops(mary)$$

CP-theory

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Example

$$
\begin{aligned}
\mathit{shops}(\mathit{john}) : 0.2 &\leftarrow \cdot \\
\mathit{shops}(\mathit{mary}) : 0.9 &\leftarrow \cdot \\
(\mathit{bought}(\mathit{spaghetti}) : 0.5) \vee (\mathit{bought}(\mathit{steak}) : 0.5) &\leftarrow \mathit{shops}(\mathit{john}) \\
(\mathit{bought}(\mathit{spaghetti}) : 0.3) \vee (\mathit{bought}(\mathit{fish}) : 0.7) &\leftarrow \mathit{shops}(\mathit{mary})
\end{aligned}
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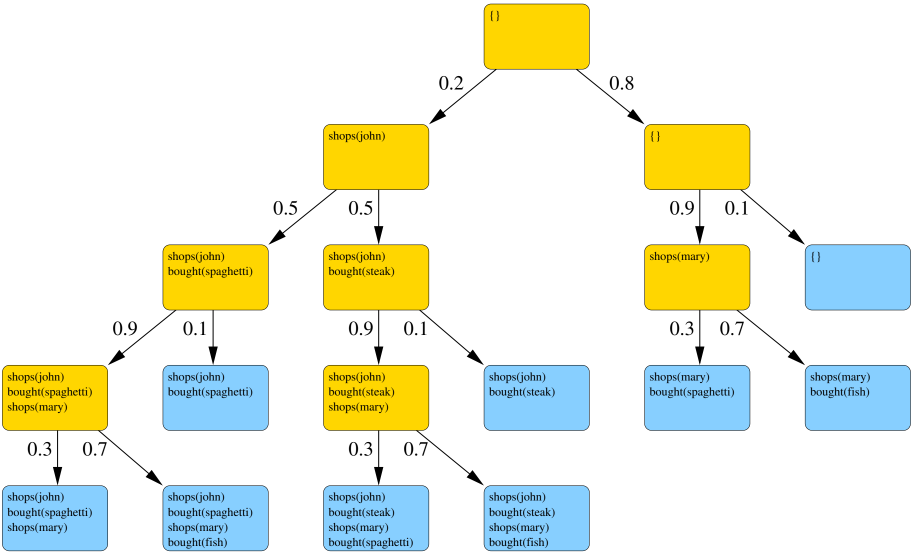
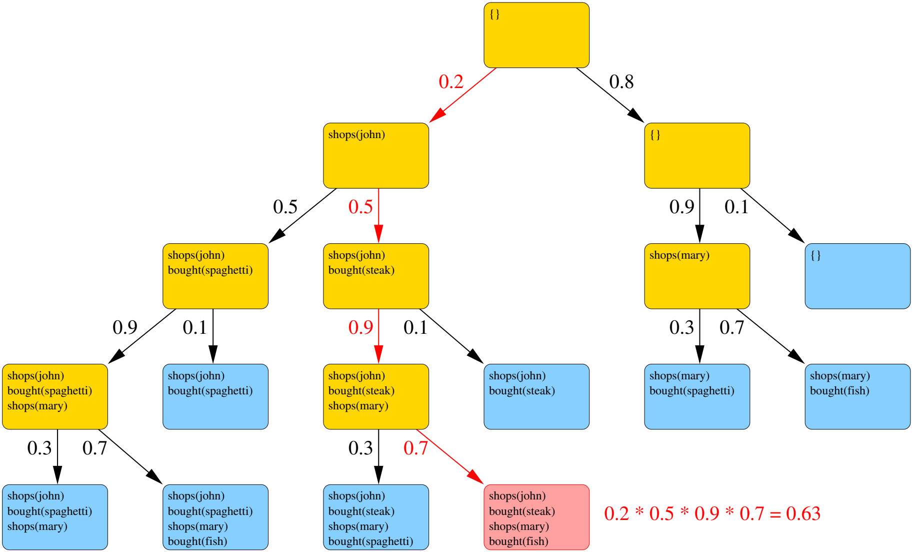
- Execution model of a CP-theory is a probabilistic process.
- Instead of definition, which is quite technical, we provide an example of an execution model.

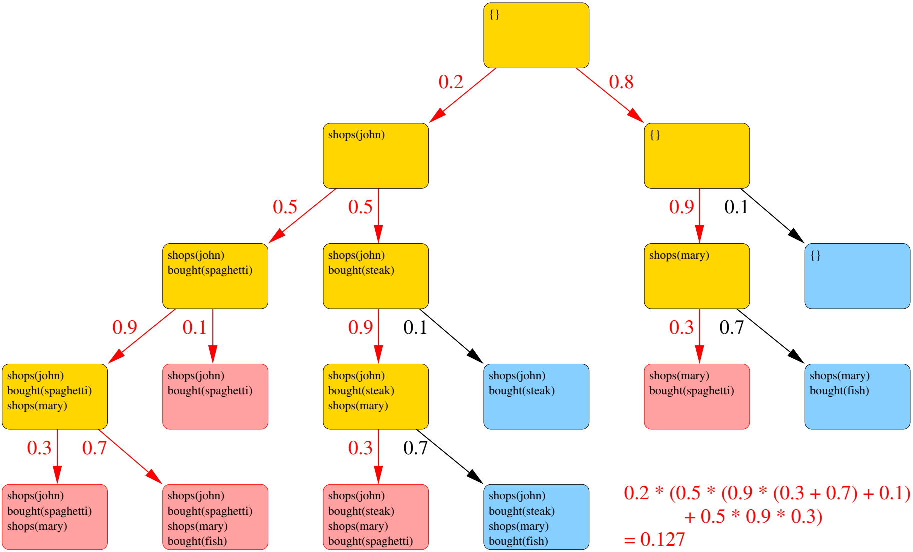
An execution model of a CP-theory



Probability of getting to a leaf



Marginal probability $P(bought(spaghetti))$



$$0.2 * (0.5 * (0.9 * (0.3 + 0.7) + 0.1) + 0.5 * 0.9 * 0.3) = 0.127$$

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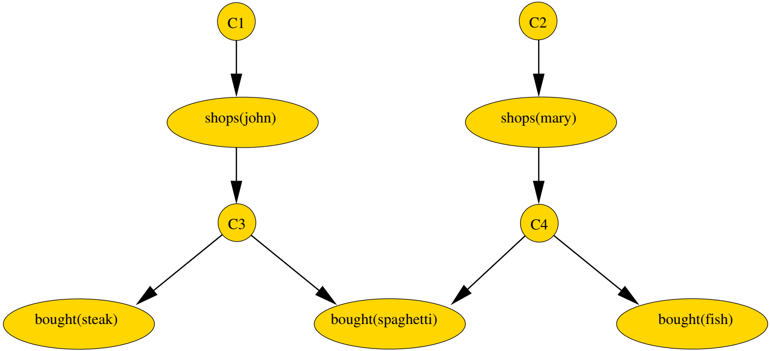
4. If an atom is in the head of a rule, an edge is created from its corresponding choice node towards the atom's node.
5. If a literal is in the body of a rule, an edge is created from the literal's atom node towards the rules choice node.

The BN of a CP-theory - Example



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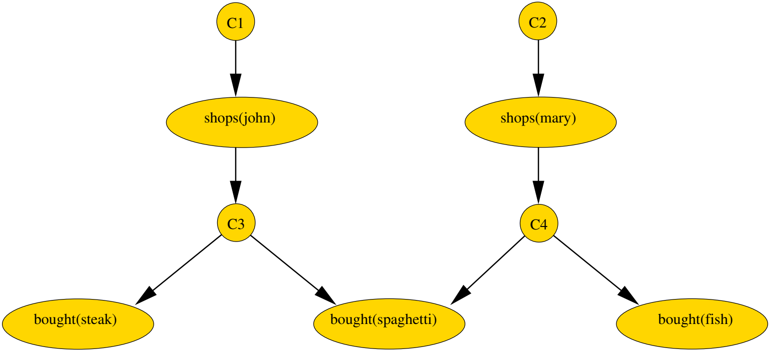
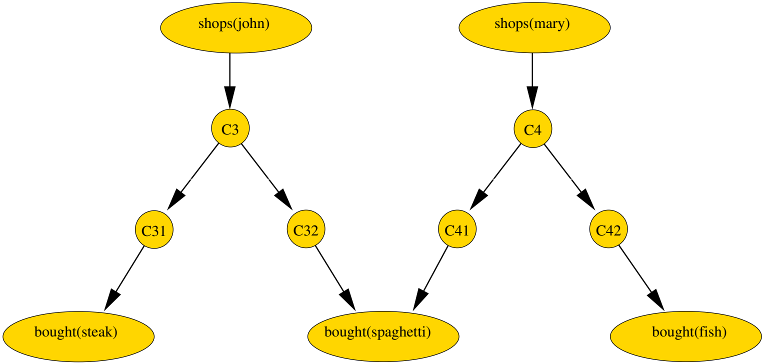


For complete specification of the BN
(including numerical parameters in conditional probability tables)
see `shopping-example-orig.net` in Hugin Light.

Modified BN of a CP-theory - Example



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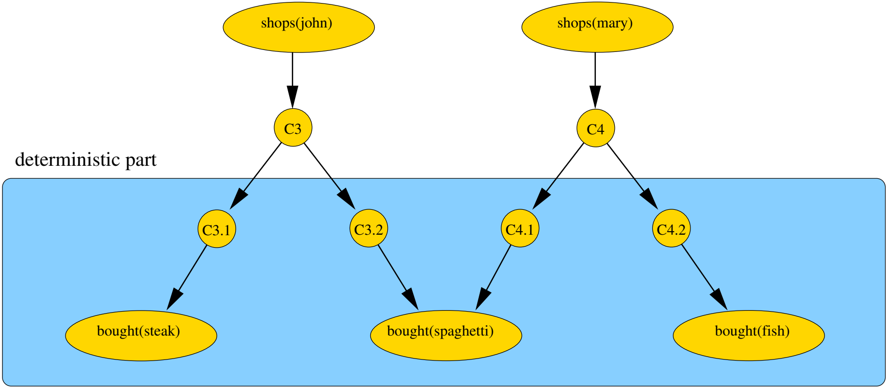

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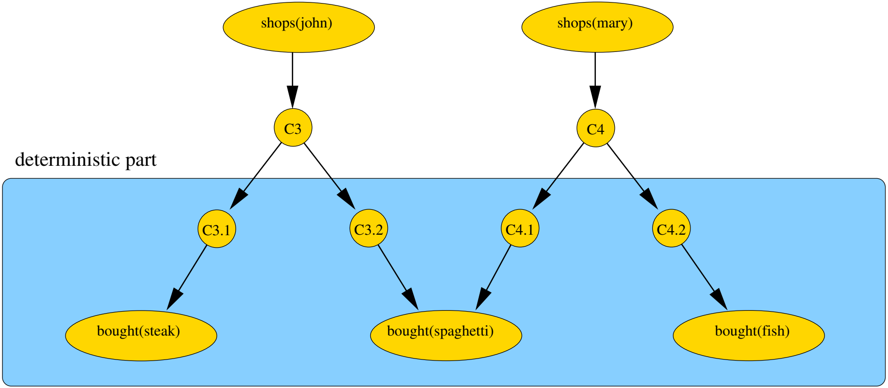
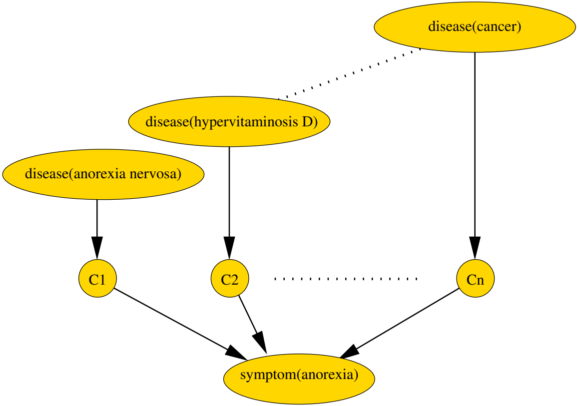


For modified BN and computations with this BN see `shopping-example.net` in Hugin Light. Compare behavior with evidence $bought(spaghetti)$ and $\neg bought(spaghetti)$.

A part of a BN with the OR function



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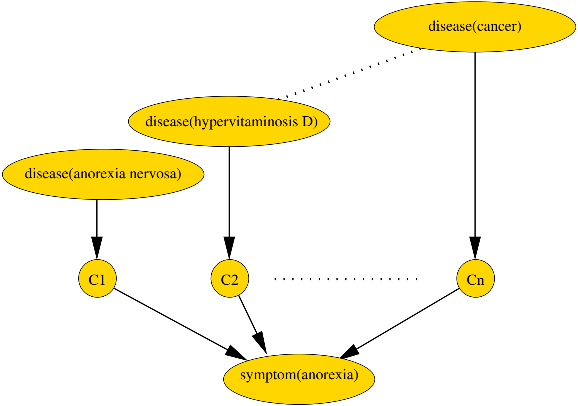

A well-known model often called noisy-OR.

Efficient probabilistic inference in BNs containing noisy-OR

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- Rank-one decomposition (Savický & Vomlel, 2007).

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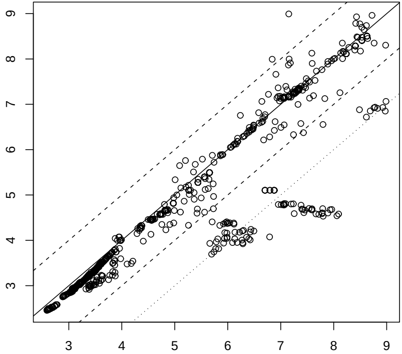

Decadic logarithm scale is used.

Current and future work

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- Graph triangulation methods for BN2O networks.

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- The rank-one decomposition of tables of ℓ-out-of-k functions.

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Future work

- Aproximate inference based on rank-one decomposition.
- An application in medicine.