Troubleshooting: NP-hardness and solution methods

Marta Sochorová and Jiří Vomlel

marta@cs.auc.dk, jirka@cs.auc.dk

Research Unit of Decision Support Systems

Aalborg University, Denmark

- An example: Light Print Problem
- Troubleshooting strategy
- A criteria for strategy evalution: Expected Cost of Repair
- NP-hardness of troubleshooting problem
- Heuristic methods providing optimal strategy
- Suboptimal strategies: BATS troubleshooter
- Comparisons of BATS strategies with optimal ones

Your **trouble**:	"The page that came out of your printer is light."
Our **trouble-shooter**:	"Perform these steps that will help you solve the trouble."

Problem description:

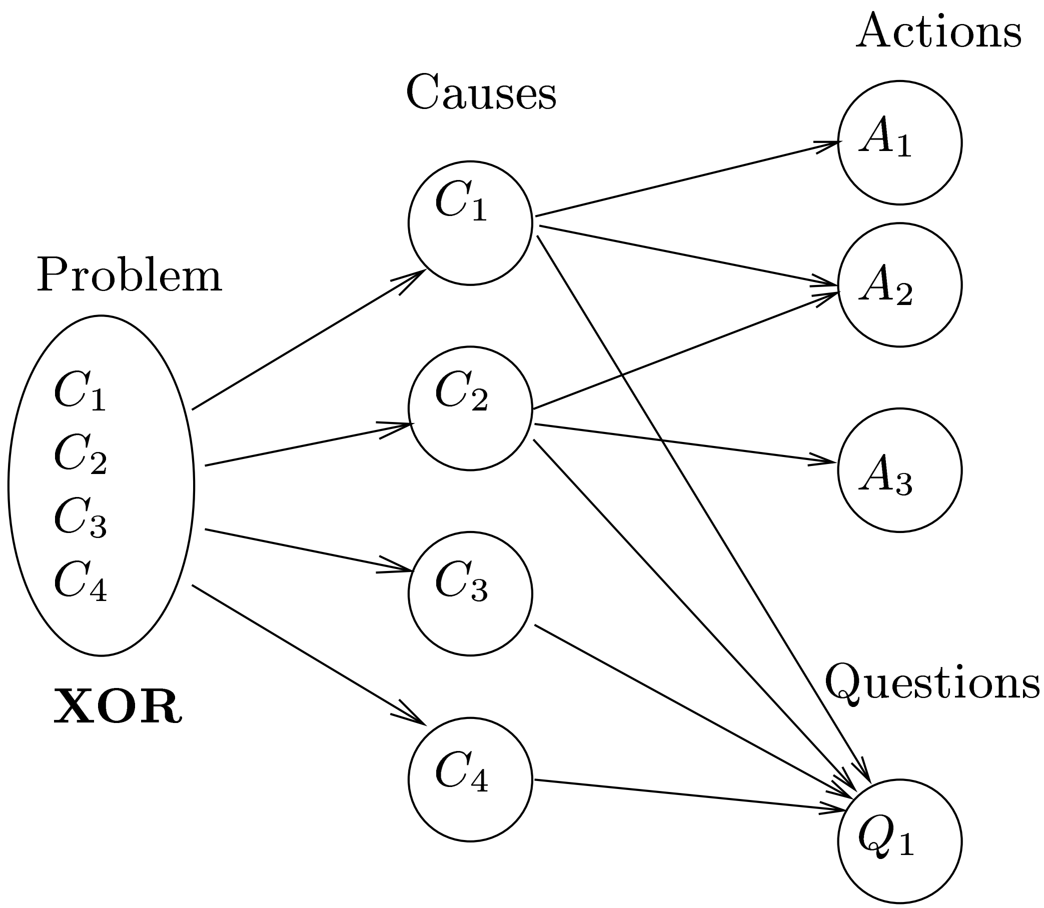
- *problem causes* $C \in \mathcal{C}$
- *actions* $A \in \mathcal{A}$ - troubleshooting steps that may solve the problem
- *questions* $Q \in \mathcal{Q}$ - troubleshooting steps that help identify the problem cause.
- every action and question has assigned a cost:

  c_A ... cost of an action A

  c_Q ... cost of a question Q

Causes of light print	$p(C_i)$
C_1: `Distribution problem`	0.4
C_2: `Defective toner`	0.3
C_3: `Corrupted dataflow`	0.2
C_4: `Wrong driver setting`	0.1

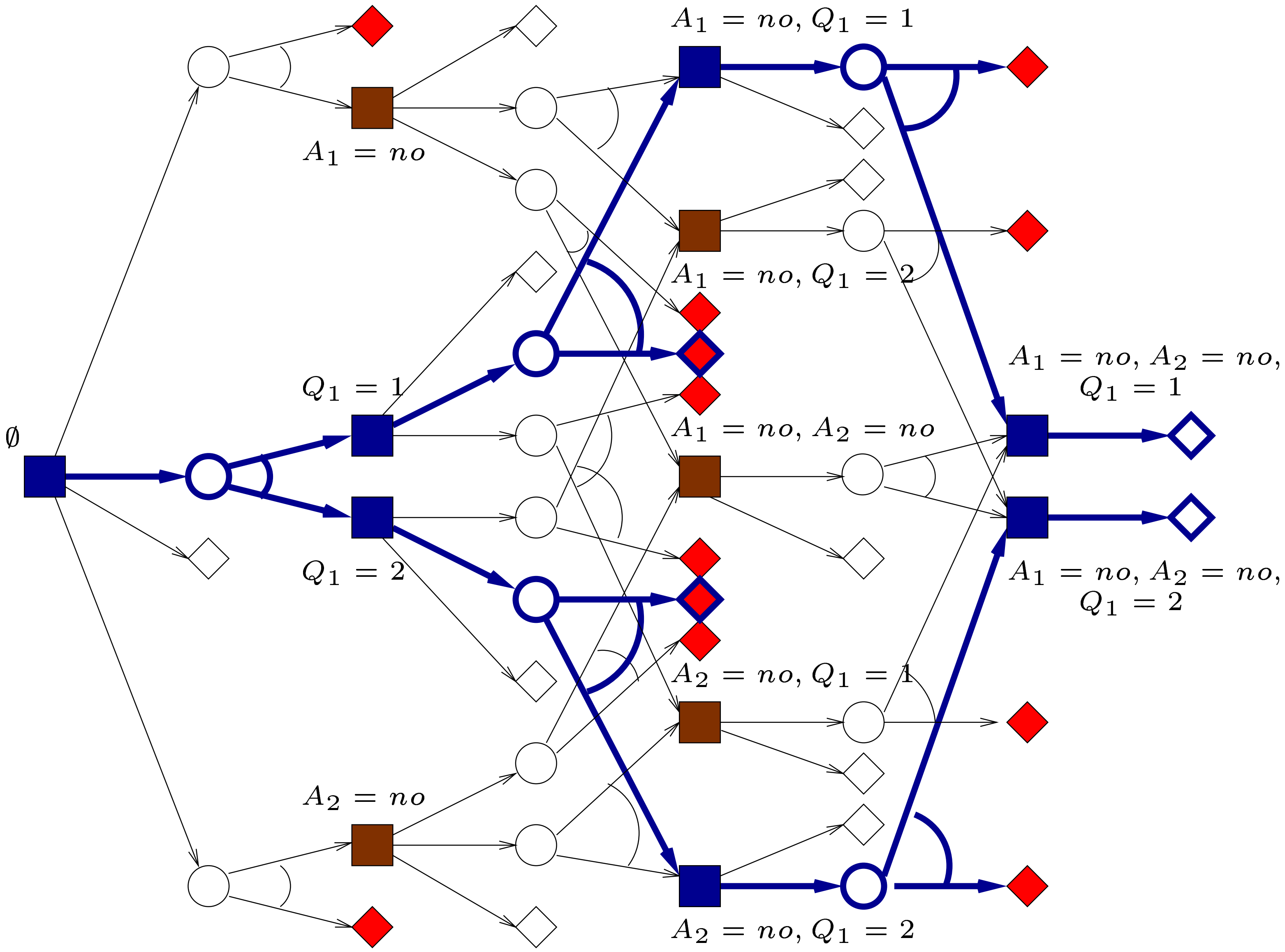
Actions and questions	c_i
A_1: `Remove, shake and reseat toner`	5
A_2: `Try another toner`	15
A_3: `Cycle power`	1
Q_1: `Is the printer configuration page printed light?`	2


Problem
C_1
C_2
C_3
C_4
XOR
Causes
C_1
C_2
C_3
C_4
Actions
A_1
A_2
A_3
Questions
Q_1

- for every action A_i and for every parent cause C_j an expert provides a CPT for $p(A_i = yes|C_j)$
- for every answer q_k to every question Q_k and for every parent cause C_j the expert provides a CPT for $p(Q_k = q_k|C_j)$

C_j	$p(A_2 = yes\|C_j)$
C_1	0.9
C_2	0.9
C_3	-
C_4	-

C_j	$p(Q_1 = yes\|C_j)$
C_1	1
C_2	1
C_3	0
C_4	0



$A_1 = no, Q_1 = 1$
$A_1 = no$
$A_1 = no, Q_1 = 2$
$A_1 = no, A_2 = no,$
$Q_1 = 1$
$Q_1 = 1$
$\emptyset$
$A_1 = no, A_2 = no$
$Q_1 = 2$
$A_1 = no, A_2 = no,$
$Q_1 = 2$
$A_2 = no, Q_1 = 1$
$A_2 = no$
$A_2 = no, Q_1 = 2$

A strategy may terminate:

- by giving up (e.g. if there are no further steps left)

  a penalty function $c(\mathbf{e}_\ell)$ applies

  can be interpreted as a cost of calling service

- by solving the problem: $c(\mathbf{e}_\ell) \stackrel{\text{def}}{=} 0$

$$\text{Node } n \quad \mapsto \quad \mathbf{e}_n = \left\{ \begin{array}{l} (A = yes/no)_{A \,\in\, \{performed\ actions\}}\,, \\ (Q = yes/no)_{Q \,\in\, \{performed\ questions\}} \end{array} \right\}$$

$\mapsto \quad p(\mathbf{e}_n)$ ... probability of getting to node n

$\mapsto \quad t(\mathbf{e}_n)$ ... total cost of actions and questions perfomed (to get to node n)

$$ECR(\mathbf{s}) \quad = \quad \sum_{\ell \in \{terminal\ nodes\ of\ \mathbf{s}\}} p(\mathbf{e}_\ell) \cdot [\ t(\mathbf{e}_\ell) + c(\mathbf{e}_\ell)\ ]$$

Strategy	ECR
A_3	$p(A_3 = yes) \cdot (c_{A_3} + 0)$
	$+\ p(A_3 = no) \cdot (c_{A_3} + c_{CS})$
A_3, A_2	$p(A_3 = yes) \cdot (c_{A_3} + 0)$
	$+\ p(A_3 = no, A_2 = yes) \cdot (c_{A_3} + c_{A_2} + 0)$
	$+\ p(A_3 = no, A_2 = no) \cdot (c_{A_3} + c_{A_2} + c_{CS})$
$Q \begin{cases} A_1 \\ A_3 \end{cases}$	$p(Q = yes, A_1 = yes) \cdot (c_Q + c_{A_1} + 0)$
	$+\ p(Q = yes, A_1 = no) \cdot (c_Q + c_{A_1} + c_{CS})$
	$+\ p(Q = yes, A_2 = yes) \cdot (c_Q + c_{A_2} + 0)$
	$+\ p(Q = yes, A_2 = no) \cdot (c_Q + c_{A_2} + c_{CS})$

Optimal strategy $\mathbf{s}^\star$

$$\Longleftrightarrow$$

$$\mathbf{s}^\star \quad = \quad \arg\min_{\mathbf{s} \in \{all \ \ possible \ \ strategies\}} ECR(\mathbf{s})$$

Theorem 1: *Suppose decision-theoretic troubleshooting problem with fixed costs and dependent actions. The decision whether there exists a troubleshooting sequence with $ECR \leq K$ for a given constant K is NP-complete problem for both single fault assumption and independent faults.*

Proof: The problem is NP: if we guess a good sequence we calculate ECR and compare whether $ECR \leq K$. It is polynomial time to calculate ECR of a sequence.

The problem is NP-hard: We reduce the Exact cover by 3-sets to troubleshooting.

> **Exact cover by 3-sets:** We are given a family $F = \{S_1, \ldots, S_n\}$ of subsets of a set U, such that $|U| = 3m$ for some integer m, and $|S_i| = 3$ for all i. We are asked if there are m sets in F that are disjoint and have U as their union.

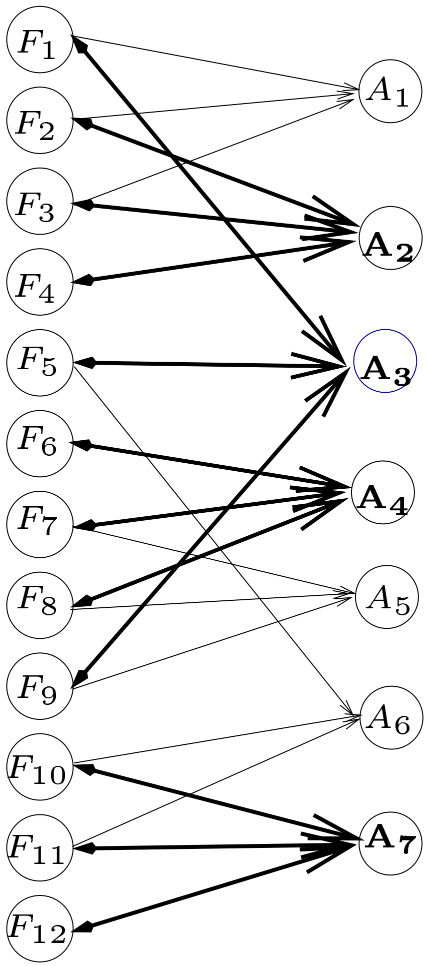
The proof of NP-completeness is for example in Christos H. Papadimitriou. *Computational complexity.* Addison-Wesley Publishing Company, 1994.

$p(C_*)$ uniform

$p(A_j|C_i) \in \{0, 1\}$

$c_A = 1$

$c(\mathbf{e}_\ell) = 2 \cdot (m+1)^2$



$\mathbf{U = \{1, 2, 3 \cdots, 12\}}$

$S_1 = \{1, 2, 3\}$

$\mathbf{S_2 = \{2, 3, 4\}}$

$\mathbf{S_3 = \{1, 5, 9\}}$

$\mathbf{S_4 = \{6, 7, 8\}}$

$S_5 = \{7, 8, 9\}$

$S_6 = \{5, 10, 11\}$

$\mathbf{S_7 = \{10, 11, 12\}}$

The exact cover by 3-sets exists iff $ECR \leq \frac{m+1}{2}$ for some sequence.

Lemma 1: *If we have exact 3-sets cover* $V = \{S_{j_1}, \ldots, S_{j_l}\}$ *then the ECR of corresponding action sequence* $A_{j_1}, \ldots, A_{j_l}$ *(in any order) has the* $ECR = \frac{m+1}{2}$.

Proof:

- $c(\mathbf{e}_\ell) > 0$ is never applied, otherwise $ECR \geq p(C) \cdot c(\mathbf{e}_\ell) > \frac{1}{3m} \cdot 2 \cdot (m+1)^2 > \frac{m+1}{2}$.
- In any step i we address three causes, therefore the value added in the terminal node i is $p(\mathbf{e}_i) \cdot t(\mathbf{e}_i) = [3 \cdot p(C)] \cdot i = 3 \cdot \frac{1}{3m} \cdot i = \frac{i}{m}$.

Therefore: $$ECR(A_{j_1}, \ldots, A_{j_l}) = \sum_{i=1}^{m} \frac{i}{m} = \frac{(m+1) \cdot m}{2 \cdot m} = \frac{m+1}{2}.$$

Lemma 2: $ECR(S) \geq \frac{m+1}{2}$ for any sequence is S. If two actions in the sequence address the same cause then $ECR(S) > \frac{m+1}{2}$.

$$T \left(\begin{array}{c} p(A|C) \in \{0,1\},\, p(C) = \frac{1}{|C|} \\ \textit{dependent actions} \\ ECR < K \end{array} \right) \textit{ is NP-complete.}$$

Consequence: *Any extension of this troubleshooting is NP-hard*, e.g.

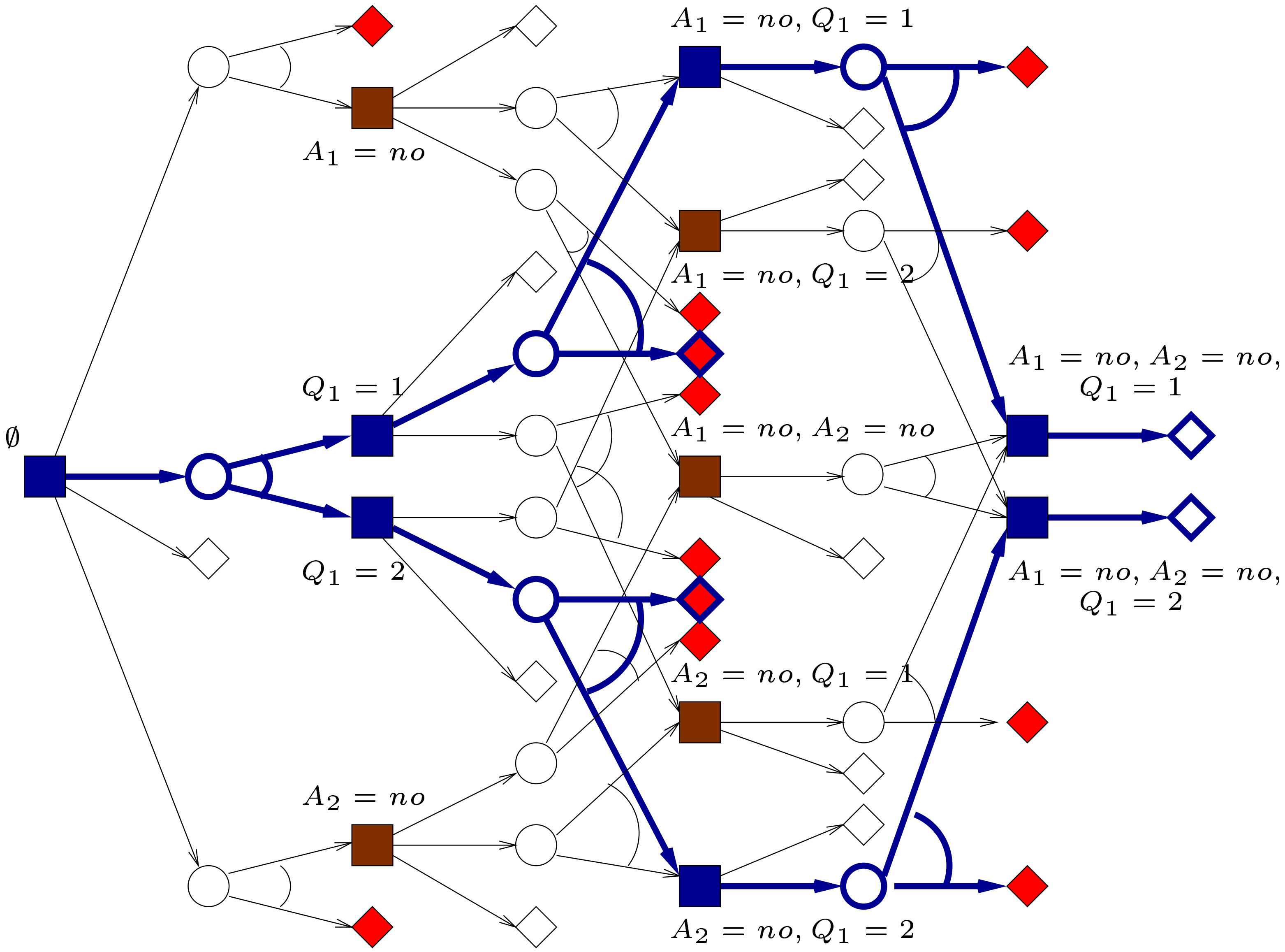
- finding a sequence with minimal ECR is NP-hard
- troubleshooting with dependent actions and questions is NP-hard

polynomial ?

$$T\begin{pmatrix}\text{indep}\\ \text{actions}\end{pmatrix} \qquad T\begin{pmatrix}\text{one or two causes per action}\\ p(A|C)\in\{0,1\}\\ p(C)=\frac{1}{|C|},\ \text{Cost}=1\end{pmatrix} \preceq \begin{matrix}\text{MAXIMAL}\\ \text{MATCHING}\end{matrix} \qquad T\begin{pmatrix}\text{one or two}\\ \text{causes per action}\\ \text{general}\end{pmatrix}$$

NP-complete NPO-complete

$$\begin{matrix}\text{EXACT COVER}\\ \text{BY 3-SETS}\end{matrix} \preceq T\big(\text{ ECR} \leq \text{K }\big) \preceq T(\text{ min ECR }) \preceq \begin{matrix}\text{TRAVELING}\\ \text{SALESMAN PROBLEM}\end{matrix}$$


$A_1 = no, Q_1 = 1$
$A_1 = no$
$A_1 = no, Q_1 = 2$
$A_1 = no, A_2 = no,$
$Q_1 = 1$
$Q_1 = 1$
$A_1 = no, A_2 = no$
$\emptyset$
$Q_1 = 2$
$A_1 = no, A_2 = no,$
$Q_1 = 2$
$A_2 = no, Q_1 = 1$
$A_2 = no$
$A_2 = no, Q_1 = 2$

- $\mathbf{s}^\star(\mathbf{e}_n)$ ... the subtree of optimal strategy $\mathbf{s}^\star$ rooted at node n
- Observe

$$\begin{aligned} \mathbf{s}^\star(A_1 = no, A_2 = no, Q_1 = yes) &\equiv \\ \mathbf{s}^\star(A_2 = no, Q_1 = yes, A_1 = no) &\equiv \\ \ldots \quad \text{any permutation of } \mathbf{e}_n & \end{aligned}$$

- Storage of $\mathbf{s}^\star(\mathbf{e}_n)$ for explored subtrees $\Longrightarrow$ reduction in search complexity:

$$\mathcal{O}((n+m)!) \longrightarrow \mathcal{O}(2^{n+m})$$

The goal:
$\widehat{ECR}(\mathbf{e}_n)$ ... an estimate of Expected Cost of Repair of strategy $\mathbf{s}^\star(\mathbf{e}_n)$ such that

$$\widehat{ECR}(\mathbf{e}_n) \leq ECR(\mathbf{e}_n),$$

so that it is an **optimistic** heuristic.

- For every $C_i \in \mathcal{C}$:
  $\mathbf{s}_{\mathbf{e}_n}^{C_i}$ denotes optimal strategy for given $\mathbf{e}_n \cup C_i = yes$.
- Define

$$\widehat{ECR}(\mathbf{e}_n) \;=\; \sum_{C_i \in \mathcal{C}} p(C_i = yes \mid \mathbf{e}_n) \cdot ECR(\mathbf{s}_{\mathbf{e}_n}^{C_i} \mid \mathbf{e}_n \cup C_i = yes)$$

- For every cause C_i the actions that may solve the problem (i.e. such that $P(A = yes \mid C_i = yes) > 0$) are ordered according to

$$\frac{p(A = yes \mid C_i = yes)}{c_A}$$

(There are usually only few such actions for every cause).

- The cause is known therefore $\forall A_j, A_k \in \mathcal{A} : \quad A_j \perp\!\!\!\perp A_k \mid C_i = yes$ and the sequence of actions ordered according to p/c ratio is optimal strategy $\mathbf{s}_{\mathbf{e}_n}^{C_i}$ (S. Srinivas, 1995).

- $p(A = yes \mid C_i = yes)$ can be read from the original model.

Observe: an update of the model is necessary only for $p(C_i|\mathbf{e}_n)$.

No other expensive computations are required!

The algorithm performs a depth first search with pruning:

- Store the temporary best $ECR'(\mathbf{e}_n)$

- If

  $$C_S + \sum_{outcomes} P(S = outcome|e) \cdot \widehat{ECR}(e \cup S = outcome) \geq ECR'(\mathbf{e}_n)$$

  then prune the branch starting with step S.

Since the applied estimate $\widehat{ECR}$ is optimistic the optimum is guaranteed.

In spite of reasonable quality of the ECR estimate (difference between true and estimated ECR was $4\% - 8\%$), the pruning rate was only about 20%.

$A^\star$ for AND/OR graphs described in:
J. Pearl, Heuristics: intelligent search strategies for computer problem solving, 1984

- All not expanded neighbors of frontier nodes are evaluated using the heuristic function $\widehat{ECR}$.
- All partial strategies are evaluated by ECR while for the not expanded neighbors of frontier nodes the $\widehat{ECR}$ value is used
- From all partial strategies the cheapest one is chosen.
- A frontier node of the cheapest strategy is expanded (different approaches).

The first fully expanded strategy is optimum.

BATS troubleshooter:

- Developed in the Laboratory for Normative Systems, within a joint project of the Hewlett-Packard and Aalborg University.
- Exploits several heuristics based on the p/c ratio.
- Described in:
  C. Skaaning, F. V. Jensen, and U. Kjærulff.
  Printer troubleshooting using Bayesian Networks. In *the Thirteenth International Conference on Industrial & Engineering Applications of AI & Expert Systems*, 2000.

Problem	$\|\mathcal{A}\|$	$\|\mathcal{Q}\|$	OPTIM	BATS	LBE	P/C
53	6	2	**433.238**	443.305	501.625	444.544
Tray	9	3	**129.214**	129.214	131.585	155.096
Overrun	11	3	**106.204**	112.456	117.377	116.801
Load	12	3	**38.3777**	38.4229	42.6062	43.0535
Pjam	13	4	**124.323**	124.365	299.415	300.855
Scatter	14	4	**115.410**	115.862	324.38	236.578
NotDupl	9	9	**70.6740**	73.5984	77.3768	121.098
Spots	16	5	**161.385**	162.246	863.362	286.749
MIO1	10	10	**250.452**	253.310	355.943	479.956