Two applications of Bayesian networks

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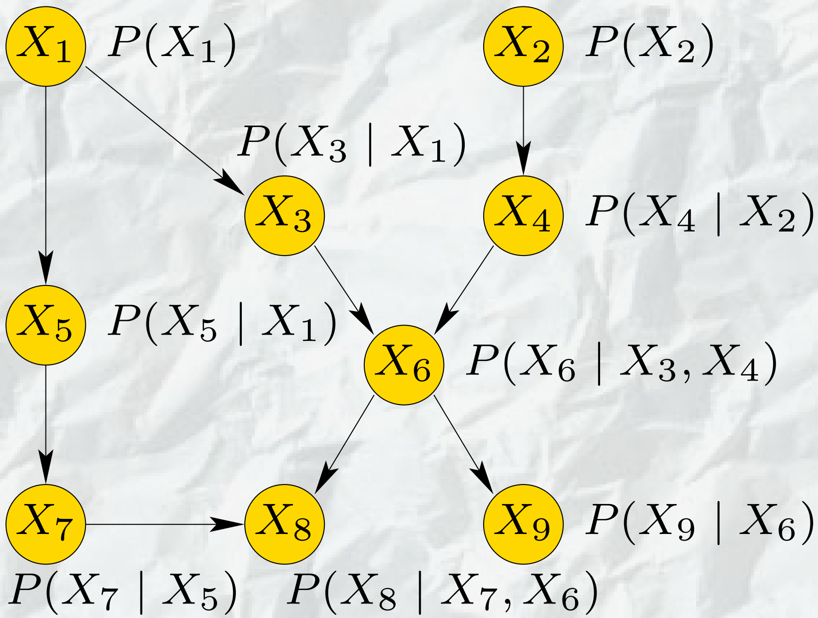
Contents:

- Bayesian networks as a model for reasoning with uncertainty
- Building probabilistic models
- Building “good” strategies using the models
- Application 1: Adaptive testing
- Application 2: Decision-theoretic troubleshooting

An example of a Bayesian network:

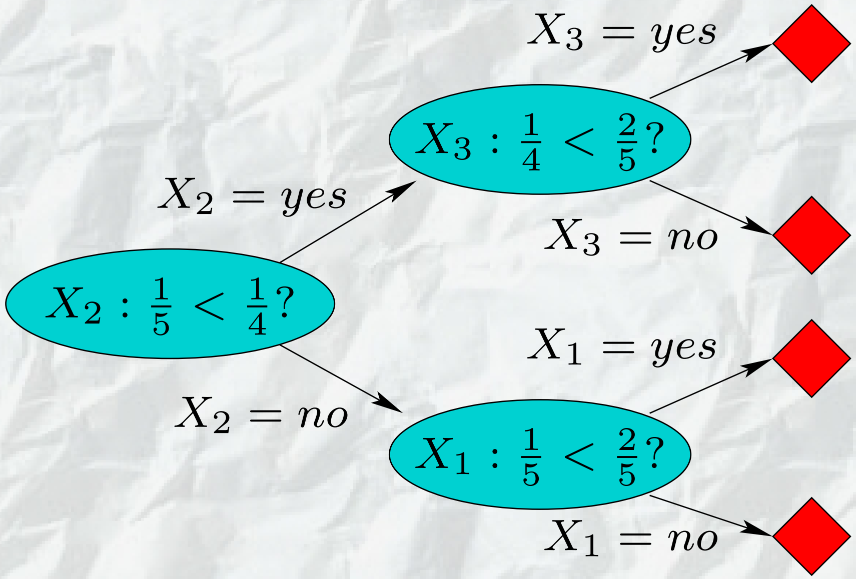


Building Bayesian network models

three basic approaches

- Discussions with domain experts: expert knowledge is used to get the structure and parameters of the model
- A dataset of records is collected and a machine learning method is used to to construct a model and estimate its parameters.
- A combination of previous two: e.g. experts helps with the stucture, data are used to estimate parameters.

An example of a strategy:



X_3 is more difficult question than X_2 which is more difficult than X_1.

Building strategies using the models

For all terminal nodes $\ell \in \mathcal{L}(\mathbf{s})$ of a strategy $\mathbf{s}$ we define:

- steps that were performed to get to that node (e.g. questions answered in a certain way). It is called collected evidence $\mathbf{e}_\ell$.
- Using the probabilistic model of the domain we can compute probability of getting to a terminal node $P(\mathbf{e}_\ell)$.
- Also during the process, when we have collected certain evidence $\mathbf{e}$ we can update the probability of getting to a terminal node, which now corresponds to conditional probability $P(\mathbf{e}_\ell)$

Building strategies using the models

For all terminal nodes $\ell \in \mathcal{L}(\mathbf{s})$ of a strategy $\mathbf{s}$ we have also defined:

- an evaluation function $f : \cup_{\mathbf{s} \in \mathcal{S}} \mathcal{L}(\mathbf{s}) \mapsto \mathbb{R}$.

For each strategy we can compute:

- expected value of the strategy:

$$E_f(\mathbf{s}) = \sum_{\ell \in \mathcal{L}(\mathbf{s})} P(\mathbf{e}_\ell) \cdot f(\mathbf{e}_\ell)$$

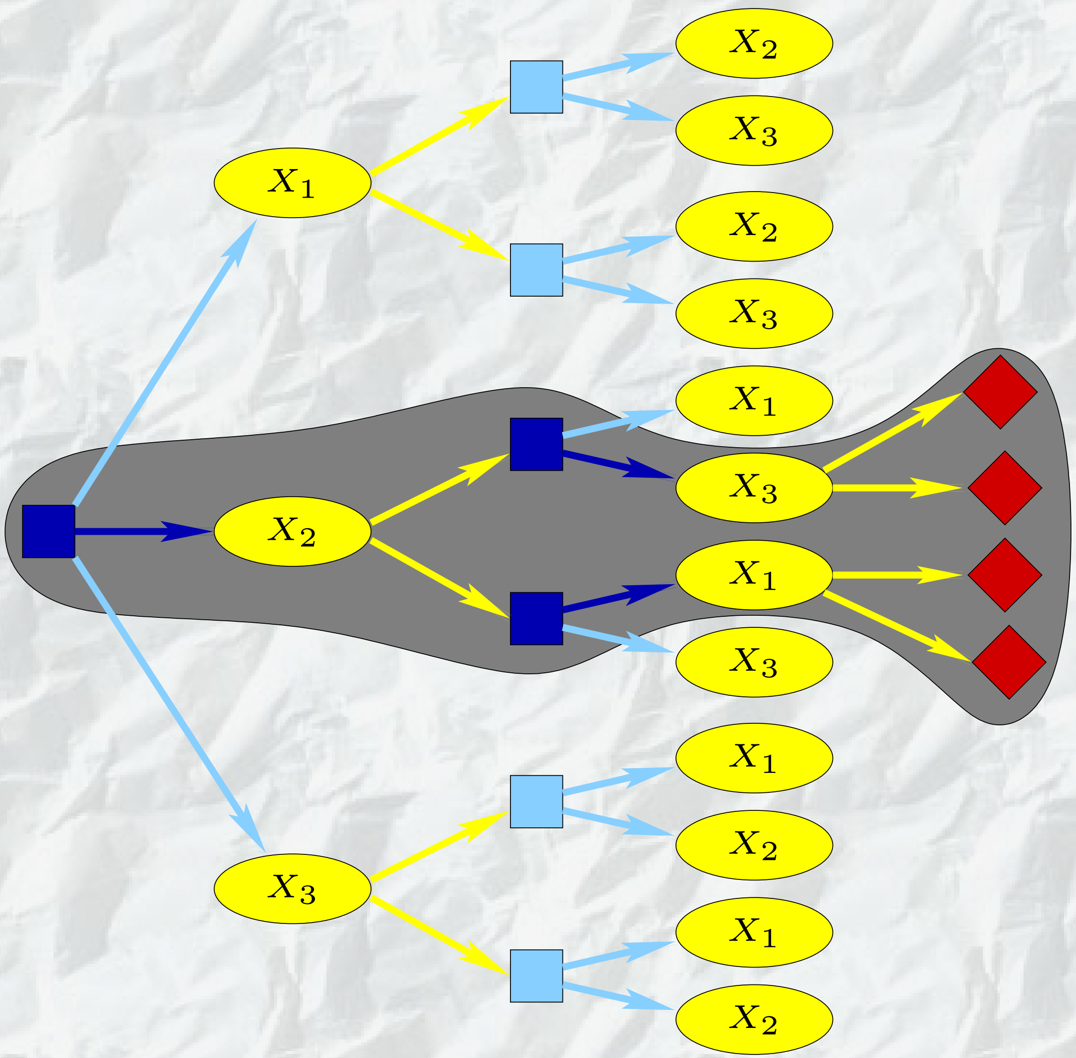
The goal:

- find a strategy that maximizes (minimizes) its expected value

Using entropy as an information measure

“The lower the entropy of a probability distribution the more we know.”

$$H\left(P(\mathbf{S})\right) \;=\; -\sum_{\mathbf{s}} P(\mathbf{S}=\mathbf{s}) \cdot \log P(\mathbf{S}=\mathbf{s})$$



Entropy in node n

$$H(\mathbf{e}_n) = H(P(\mathbf{S} \mid \mathbf{e}_n))$$

Expected entropy at the end of test $\mathbf{t}$

$$E_H(\mathbf{t}) = \sum_{\ell \in \mathcal{L}(\mathbf{t})} P(\mathbf{e}_\ell) \cdot H(\mathbf{e}_\ell)$$

$\mathcal{T}$ ... the set of all possible tests (e.g. of a given length)

A test $\mathbf{t}^\star$ is **optimal** iff

$$\mathbf{t}^\star = \arg\min_{\mathbf{t} \in \mathcal{T}} E_H(\mathbf{t}) \ .$$

Application 1: Adaptive test of basic operations with fractions

Examples of tasks:

$$T_1: \quad \left(\frac{3}{4}\cdot\frac{5}{6}\right)-\frac{1}{8} \quad = \quad \frac{15}{24}-\frac{1}{8}=\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

$$T_2: \quad \frac{1}{6}+\frac{1}{12} \quad = \quad \frac{2}{12}+\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$$

$$T_3: \quad \frac{1}{4}\cdot 1\frac{1}{2} \quad = \quad \frac{1}{4}\cdot\frac{3}{2}=\frac{3}{8}$$

$$T_4: \quad \left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot\left(\frac{1}{3}+\frac{1}{3}\right) \quad = \quad \frac{1}{4}\cdot\frac{2}{3}=\frac{2}{12}=\frac{1}{6}\ .$$

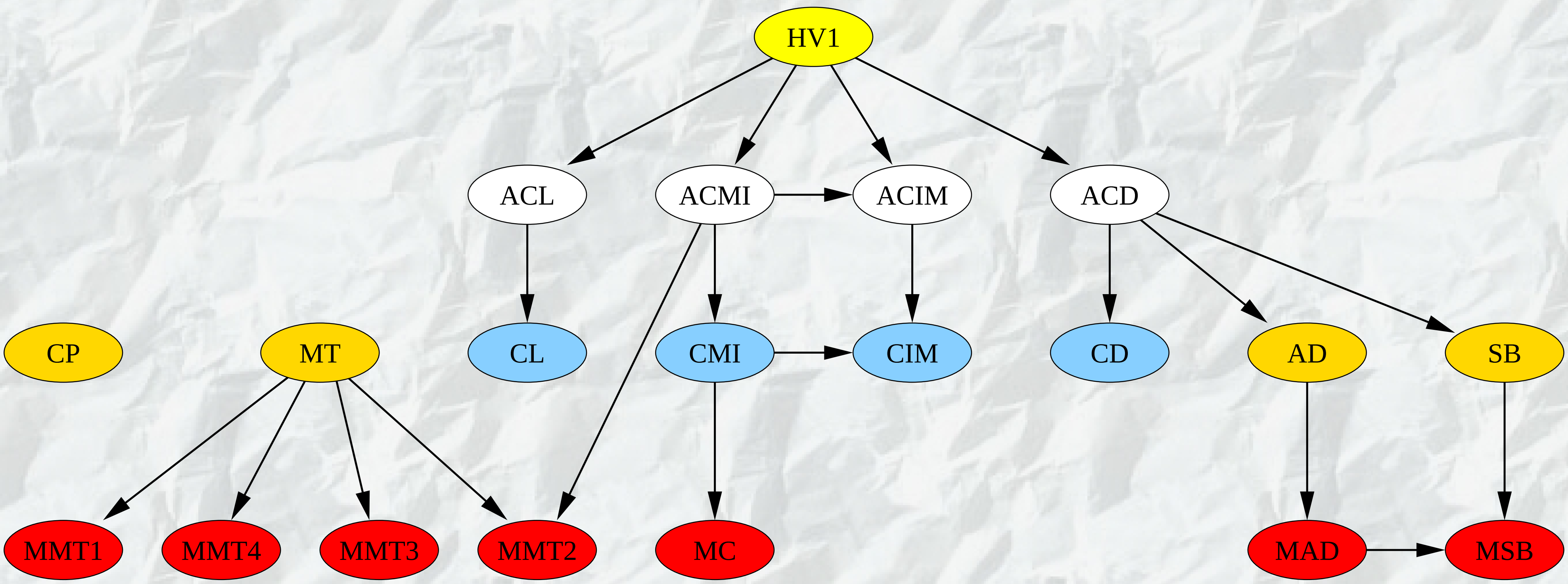
Elementary and operational skills

CP	Comparison (common numerator or denominator)	$\frac{1}{2} > \frac{1}{3}, \quad \frac{2}{3} > \frac{1}{3}$
AD	Addition (comm. denom.)	$\frac{1}{7} + \frac{2}{7} = \frac{1+2}{7} = \frac{3}{7}$
SB	Subtract. (comm. denom.)	$\frac{2}{5} - \frac{1}{5} = \frac{2-1}{5} = \frac{1}{5}$
MT	Multiplication	$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$
CD	Common denominator	$\left(\frac{1}{2}, \frac{2}{3}\right) = \left(\frac{3}{6}, \frac{4}{6}\right)$
CL	Cancelling out	$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}$
CIM	Conv. to mixed numbers	$\frac{7}{2} = \frac{3 \cdot 2+1}{2} = 3\frac{1}{2}$
CMI	Conv. to improp. fractions	$3\frac{1}{2} = \frac{3 \cdot 2+1}{2} = \frac{7}{2}$

Misconceptions

Label	Description	Occurrence
MAD	$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$	14.8%
MSB	$\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$	9.4%
MMT1	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a \cdot c}{b}$	14.1%
MMT2	$\frac{a}{b} \cdot \frac{c}{b} = \frac{a+c}{b \cdot b}$	8.1%
MMT3	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$	15.4%
MMT4	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b+d}$	8.1%
MC	$a\frac{b}{c} = \frac{a \cdot b}{c}$	4.0%

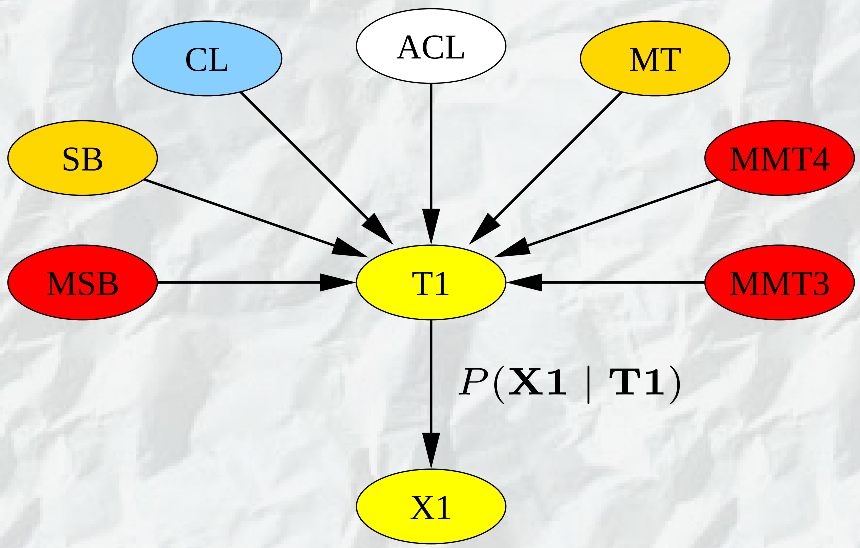
Student model

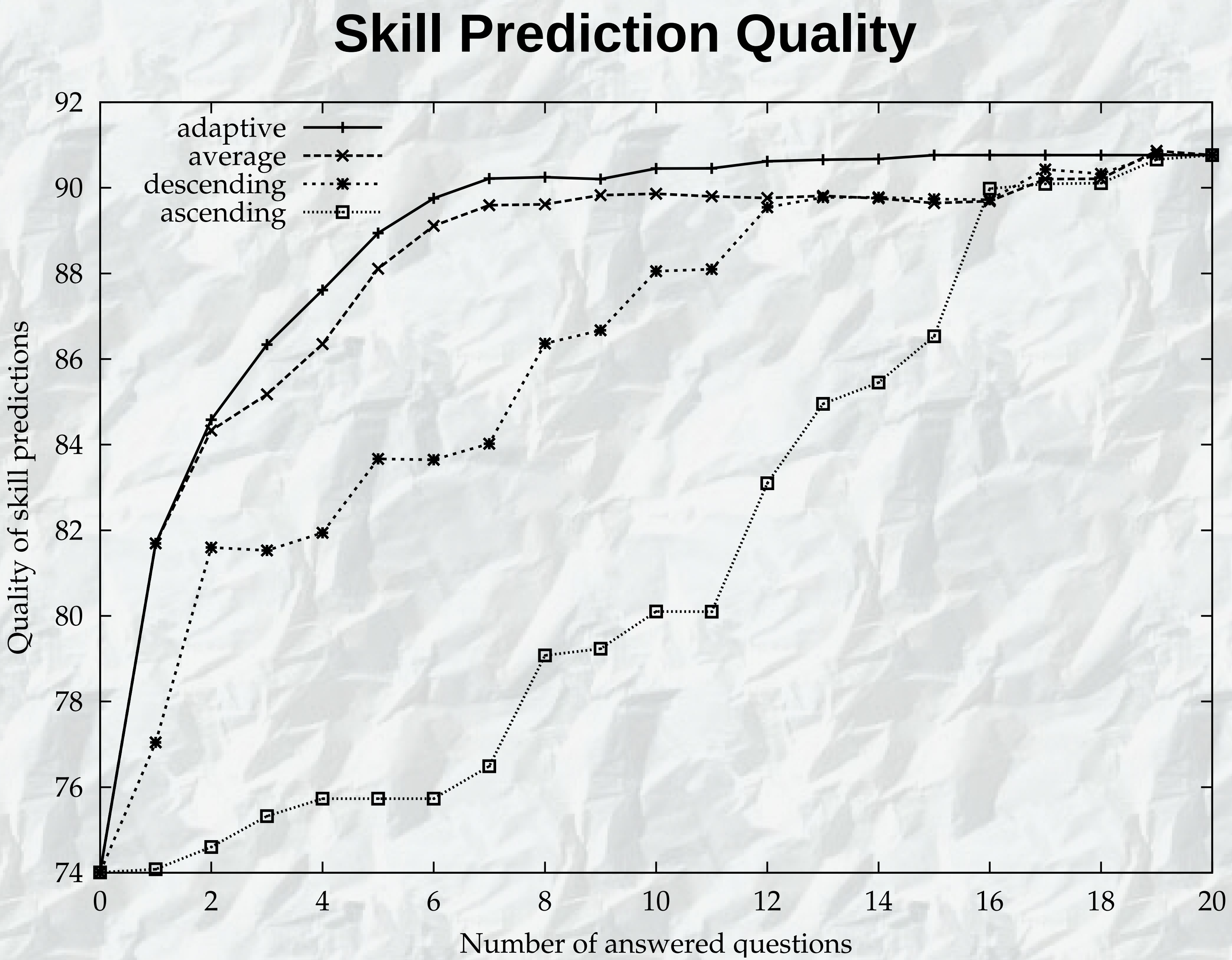


Evidence model for task $T1$

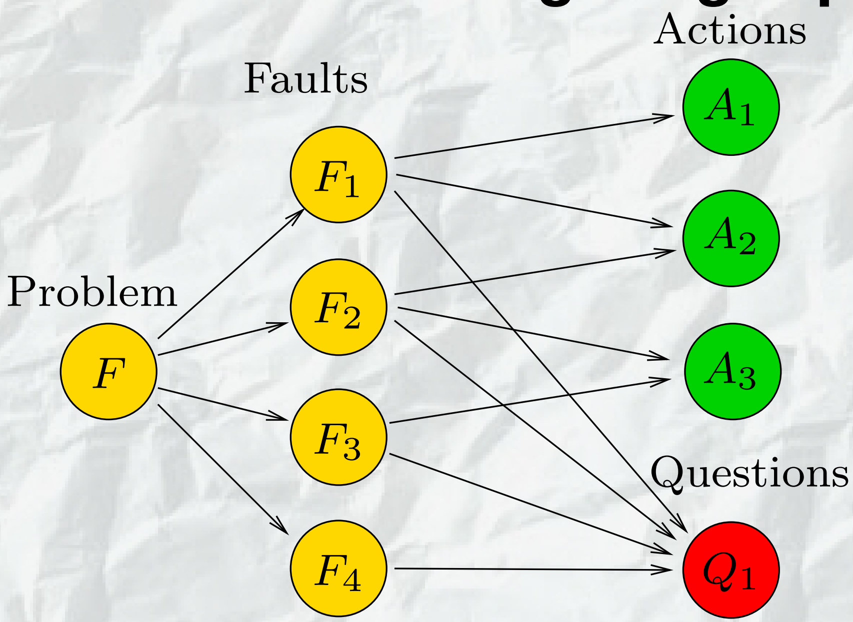
$$\left(\frac{3}{4}\cdot\frac{5}{6}\right)-\frac{1}{8}=\frac{15}{24}-\frac{1}{8}=\frac{5}{8}-\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

$$T1 \quad \Leftrightarrow \quad MT \;\&\; CL \;\&\; ACL \;\&\; SB \;\&\; \neg MMT3 \;\&\; \neg MMT4 \;\&\; \neg MSB$$



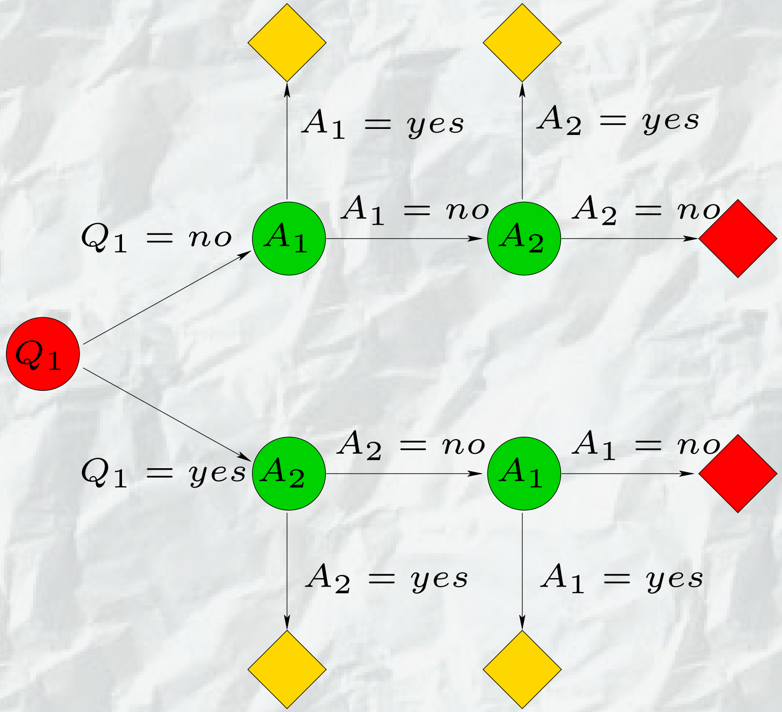

Skill Prediction Quality
adaptive
average
descending
ascending
Quality of skill predictions
Number of answered questions
92
90
88
86
84
82
80
78
76
74
0
2
4
6
8
10
12
14
16
18
20

Application 2: Troubleshooting - Light print problem



- Problems: F_1 *Distribution problem*, F_2 *Defective toner*, F_3 *Corrupted dataflow*, and F_4 *Wrong driver setting*.
- Actions: A_1 *Remove, shake and reseat toner*, A_2 *Try another toner*, and A_3 *Cycle power*.
- Questions: Q_1 *Is the configuration page printed light?*

Troubleshooting strategy



The task is to find a strategy $\mathbf{s} \in \mathcal{S}$ minimising expected cost of repair

$$E_{CR}(\mathbf{s}) = \sum_{\ell \in \mathcal{L}(\mathbf{s})} P(\mathbf{e}_\ell) \cdot (\, t(\mathbf{e}_\ell) + c(\mathbf{e}_\ell) \,) \quad .$$

Going commercial...

- Hugin Expert A/S.
  software product: Hugin - a Bayesian network tool.
  http://www.hugin.com/
- Educational Testing Service (ETS)
  the world's largest private educational testing organization
  In 2000/2001 more than 3 millions students took the ETS's largest exam SAT. Research unit doing research on adaptive test using Bayesian networks: http://www.ets.org/research/
- SACSO Project
  Systems for Automatic Customer Support Operations
  - research project of Hewlett Packard and Aalborg University.
  The troubleshooter offered as DezisionWorks by Dezide Ltd.
  http://www.dezide.com/