Image registration methods: A survey

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Abstract

This paper aims to present a review of recent as well as classic image registration methods. Image registration is the process of overlaying two or more images of the same scene taken at different times, from different viewpoints and/or by different sensors. The major registration purpose is to remove or suppress geometric distortions between the reference and sensed images. The reviewed approaches are classified according to their nature (*area-based* and *feature-based*) and according to four base steps of image registration procedure: *feature detection, feature matching, mapping function design* and *image transformation and resampling*. Main contributions, advantages and drawbacks of the methods are mentioned in the paper. Problematic issues of image registration and outlook for the future research are discussed too. The major goal of the paper is to provide a comprehensive reference source for researchers involved in image registration, regardless of particular application areas.

Keywords: Image registration, feature detection, feature matching, mapping functions, resampling

1 Introduction

Image registration is the process of overlaying two or more images of the same scene taken at different times, from different viewpoints and/or by different sensors. The major registration purpose is to remove or suppress geometric distortions between the reference and sensed images, which were introduced due to different imaging conditions, and thus to bring images into geometric alignment. Image registration is a crucial step in all image analysis tasks in which the final information is gained by combining various data sources. Typically, registration is required in remote sensing (multispectral classification, image fusion, environmental monitoring, change detection, image mosaicing, weather forecasting, integrating information into GIS), in medicine (combining CT and NMR data to obtain more complete information about the patient, monitoring of tumor growth, treatment verification, comparison of the patient data with anatomical atlases), in cartography (map updating), and in computer vision (target localization, automatic quality control), to name a few.

During last decades, image acquisition devices have undergone a rapid development and growing amount and diversity of obtained images invoked the research on automatic image registration. A comperhensive survey of image registration methods was published in 1992 by Brown [1]. The intention of our article is to cover relevant approaches introduced later and in this way map current development of registration techniques. According to the database of the Institute of Scientific Information (ISI), in the last ten years more than 1000 papers were published on the image registration topic. Methods published before 1992 that became classic or introduced key ideas, which are still in use, are included too to retain the continuity and to give connected view of image registration research. We do not contemplate to go into details of particular algorithms or describe results of comparative experiments, rather we want to summarize main approaches and point out interesting parts of the registration methods.

In Section 2 aspects and problems of the image registration will be discussed. Both area-based and feature-based approaches to feature selection are described in Section 3. Section 4 reviews existing algorithms for feature matching. Methods for mapping function design are given in Section 5. Finally, Section 6 surveys various techniques for image transformation and resampling. Section 7 concludes main trends in the research on registration methods and offers the outlook for the future.

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2 Image registration methodology

Image registration, as it was mentioned above, is widely used in remote sensing, medical imaging, computer vision etc. In general, its applications can be divided into four main groups according to the manner of the image acquisition:

Different viewpoints (multiview analysis):
Images of the same scene are acquired from different viewpoints. The aim is to gain larger 2D view or 3D representation of the scanned scene.

Examples of applications: *Remote sensing* – mosaicing of images of the surveyed area. *Computer vision* – shape recovery (shape from stereo).

Different times (multitemporal analysis):
Images of the same scene are acquired at different times, often on regular basis, and possibly under different conditions. The aim is to find and evaluate changes in the scene which appeared between the image acquisitions.

Examples of applications: *Remote sensing* – monitoring of global land usage, landscape planning. *Computer vision* – automatic change detection for security monitoring, motion tracking. *Medical imaging* - monitoring of the healing therapy, monitoring of the tumor evolution.

Different sensors (multimodal analysis):
Images of the same scene are acquired by different sensors. The aim is to integrate the information obtained from different source streams to gain more complex and detailed scene representation.

Examples of applications: *Remote sensing* – fusion of information from sensors with different characteristics like SPOT images, offering better spatial resolution, Landsat TM images with better spectral resolution, or SAR images independent on cloud cover and solar illumination. *Medical imaging* – combination of sensors recording the anatomical body structure like MRI – magnetic resonance image, ultrasound or CT- computer tomography with sensors monitoring functional and metabolic body activities like PET – positron emission tomography, SPECT – single photon emission computed tomography or MRS – magnetic resonance spectroscopy. Results can be applied, for instance, in radiotherapy and nuclear medicine.

Scene to model registration:
Images of a scene and a model of the scene are registered. The model can be a computer representation of the scene, for instance maps or digital elevation models (DEM) in geographic information systems (GIS), another scene with similar content (another patient), "average" specimen, etc. The aim is to localize the acquired image in the scene/model and/or to compare them.

Examples of applications: *Remote sensing* – registration of aerial or satellite data to maps or other GIS layers. *Computer vision* – target template matching with real-time images, automatic quality inspection. *Medical imaging* – comparison of the patient image with digital anatomical atlases, specimen classification.

Due to the diversity of images to be registered and due to various types of degradations it is impossible to design a universal method applicable to all registration tasks. Every method should take into account not only the assumed type of geometric deformation between the images but also radiometric deformations and noise corruption, required registration accuracy and application-dependent data characteristics.

Nevertheless, the majority of the registration methods consists of the following four steps:

- *Feature detection.* Salient and distinctive objects (closed-boundary regions, edges, contours, line intersections, corners, etc.) are manually or, preferably, automatically detected. For further processing, these features can be represented by their point representatives (centers of gravity, line endings, distinctive points), called in the literature *control points* (CP's).

- *Feature matching.* In this step, the correspondence between the features detected in the sensed image and those detected in the reference image is established. Various feature descriptors and similarity measures along with spatial relationships among the features are used for that purpose.
- *Transform model estimation.* The type and parameters of the so-called *mapping functions*, aligning the sensed image with the reference image, are estimated. The parameters of the mapping functions are computed by means of the established feature correspondence.
- *Image resampling and transformation.* The sensed image is transformed by means of the mapping functions. Image values in non-integer coordinates are estimated by an appropriate interpolation technique.

Implementation of each registration step has its typical problems. First, we have to decide what kind of features is appropriate for the given task. The features should be distinctive objects, which are frequently spread over the images and which are easily detectable. Usually, the physical interpretability of the features is demanded. Detected feature sets in the reference and sensed images must have enough common elements, even in situations when the images do not cover exactly the same scene or when there are object occlusions or other unexpected changes. Detection methods should have good localization accuracy and should not be sensitive to the assumed image degradation. In an ideal case, the algorithm should be able to detect the same features in all projections of the scene regardless of the particular image deformation.

In the feature matching, problems caused by incorrect feature detection or by image degradations can arise. Physically corresponding features can be dissimilar due to the different imaging conditions and/or due to the different spectral sensitivity of the sensors. The choice of the feature description and similarity measure have to consider these factors. The feature descriptors should be invariant to the assumed degradations. Simultaneously, they have to be discriminable enough to be able to distinguish among different features as well as sufficiently stable not to be influenced by slight unexpected feature changes and noise. The matching algorithm in the space of invariants should be robust and efficient. Single features without corresponding counterparts in the other image should not affect its performance.

The type of the mapping functions should be chosen according to *a priori* information about the acquisition process and expected image degradations. If no *a priori* information is available, the model should be flexible and general enough to handle all possible degradations which might appear. The accuracy of the feature detection method, reliability of feature correspondence estimation and the acceptable approximation error need to be considered too. Moreover, the decision which differences between images have to be removed by registration has to be done. It is desirable not to remove the differences we are searching for if the aim is a change detection. This issue is very important and extremely difficult.

Finally, the choice of the appropriate type of resampling technique depends on the trade-off between the demanded accuracy of the interpolation and computational complexity. Nearest-neighbor or billinear interpolation are sufficient in most cases; however, some applications require more precise methods.

Because of its importance in various application areas as well as of its complicated nature, image registration has been the topic of much recent research. The historically first survey paper [2] covers mainly the methods based on image correlation. Probably the most exhaustive review of the general-purpose image registration methods is in [1]. Registration techniques applied particularly in medical imaging are summarized in [3], [4] and [5], those mainly applied in remote sensing are described and evaluated in [6], [7] and [8].

Registration methods can be categorized with respect to various criteria. The ones usually used are the application area, dimensionality of data, type and complexity of assumed image deformations, computational cost and the essential ideas of the registration algorithm. Here, the classification according to the essential ideas is chosen, considering the decomposition of the registration into the described four steps. The techniques exceeding this four-step framework are covered according to their main contribution.

3 Feature detection

Formerly, the features were objects manually selected by an expert. During an automation of this registration step, two main approaches to feature understanding have been formed.

3.1 Area-based methods

Area-based methods, sometimes called correlation-like methods or template matching [6], put emphasis rather on the feature matching than on their detection; some authors even omit the detection step or present it as merged with the matching part. These methods deal with the images without attempting to detect salient objects. The correspondence is estimated using entire images or windows of specified size, shifted across the images regardless of an image content, see e.g. [9], [10], [11].

The limitations of the area-based methods originate in their base idea. Firstly, the rectangular window, which is most often used, suits the registration of images which locally differ only by a translation. If images are deformed by more complex transformations, this type of the window is not able to cover the same parts of the scene in the reference and sensed images. Goshtabsy [12] proposed to use circular shape of the window for mutually rotated images. However, the comparability of such simple-shaped windows is violated too if more complicated geometric deformations (similarity, perspective transforms, etc.) are present between images.

Another disadvantage of the area-based methods refers to the "remarkableness" of the window content. A window containing a smooth area without any prominent details has high probability to be matched incorrectly with other smooth areas in the reference image due to its nonsaliency. The features for registration should be preferably detected in distinctive parts of the image. Windows, whose selection is not based on their content evaluation, may not have this property.

Finally, classical area-based methods like cross-correlation exploit for matching directly image intensities, without any structural analysis. Consequently, they are sensitive to the intensity changes, introduced for instance by noise, varying illumination and/or by using different sensor types.

3.2 Feature-based methods

Second approach extracts salient structures – features – in the images. Significant regions (forests, lakes, fields), lines (region boundaries, coastlines, roads, rivers) or points (region corners, line intersections, points on curves with high curvature) are understood as features. They should be distinct, spread all over the image and efficiently detectable in both images. They are expected to be stable in time to stay at fixed positions during the whole experiment.

The comparability of feature sets in the sensed and reference images is assured by invariance and accuracy of the feature detector and by the overlap criterion. In other words, the number of common elements of detected sets of features should be sufficiently high, regardless of the change of image geometry, radiometric conditions, presence of additive noise and of changes in the scanned scene. "Remarkableness" of features is implied by their definition. In contrast to the area-based methods, the feature-based ones do not work directly with image intensity values. The features represent information on higher level. This property makes feature-based methods suitable for situations when illumination changes are expected or multisensor analysis is demanded.

Region features. Region-like features can be projections of general high contrast closed-boundary regions of an appropriate size [13], [14], water reservoirs and lakes [15], [16], buildings [17], forests [18], urban areas [19] or shadows [20]. The general criterion of closed-boundary regions is prevalent. The regions are often represented by their centers of gravity, which are invariant with respect to rotation, scaling and skewing and stable under random noise and gray level variation.

Region features are detected by means of segmentation methods [21]. The accuracy of the segmentation can significantly influence the resulting registration. Goshtasby et al. [13] proposed a refinement of the segmentation process to improve the registration quality. The segmentation of the image was done iteratively together with the registration; in every iteration, the rough estimation of the object correspondence is used to tune the segmentation parameters. They claimed the subpixel accuracy of registration can be achieved.

Line features. Line features can be representations of general line segments [22], [23], [17], object contours [24], [25], [26], coastal lines, [27], [28], roads [29] or elongated anatomic structures [30] in medical imaging. Line correspondence is usually expressed by pairs of line ends or middle points.

Standard edge detection methods, like Canny detector [31] or a detector based on the Laplacian of Gaussian [32], are employed for the line feature detection. Li et al. [24] proposed to exploit the already detected features in the reference image (optical data) for detection of lines in the sensed images (SAR images with speckle noise, which is a typical degradation present in this type of data). They applied elastic contour extraction. Comparison of different operators for feature edge detection and ridge detection in multimodal medical images is presented by Maintz et al. [33], [34].

Point features. Point features group consists of methods working with line intersections [35], [36], road crossings [19], [37], centroids of water regions, oil and gas pads [38], high variance points [39], local curvature discontinuities detected using Gabor wavelets [40], [41], inflection points of curves [42], [43], local extrema of wavelet transform [44], [45], most distinctive points with respect to a specified measure of similarity [46] and corners [47], [48], [17].

The core algorithms of feature detectors in most cases follow the definitions of the "point" like line intersection, centroid of closed-boundary region or local modulus maxima of wavelet transform. Corners form specific class of features, because "to-be-a-corner" property is hard to define mathematically (intuitively, corners are understood as points of high curvature on the region boundaries). Corners are widely used as control points mainly because of their invariance to imaging geometry and because they are well perceived by a human observer.

Much effort has been spent in developing precise, robust and fast method for corner detection. A survey of corner detectors can be found in [49], [50], [51]. The latter also analyzes localization properties of the detectors.

Kitchen and Rosenfeld [52] proposed to exploit the second-order partial derivatives of the image function for corner detection. Their operator computes the curvature of a curve, formed by the intersection of the image function with the plane perpendicular to the gradient of the image function at the given point. Corners are identified as its local extrema. Several authors (Brunnström et al. [53], Zuniga and Haralick [54]) followed this approach. Dreschler and Nagel [55] searched for the local extrema of the Gaussian curvature. Beaudet [56] proposed to find corners as local extrema of Hessian determinant of the image function. However, corner detectors based on second-order derivatives of the image function are sensitive to noise.

Förstner [57] developed more robust corner detector, although time consuming, which is based on first-order derivatives only. Reputable Harris detector (also referred as Plessey detector) [58] is in fact its inverse. An application of the Förstner detector is described in [59], where it is used for the registration of dental implants images.

Smith and Brady [60] in their SUSAN accomplished good robustness to noise by avoiding the use of image derivatives. They use the size of the area of the same color as the central pixel has as the criterion. Trajkovic and Hedley [61] designed their operator utilizing the idea that the change of image intensity at the corners should be high in all directions.

Methods based on edge detection suffer from poor performance of edge detectors on corners. Mehrotra [62] introduced the idea of half edges and constructed a corner detector by applying directional masks. Würtz and Lourens [63] used a biological motivation and modeled end-stop cells of the visual cortex. Lee et al. [64] employed a wavelet transform for the corner detection. Rohr et al. designed corner detectors, even for 3-D data, allowing user interaction [65]. Most recently, Zitová et al. [66] proposed a parametric corner detector, which does not employ any derivatives and which was designed to handle blurred and noisy data.

The number of detected points can be very high. This increases the computational time necessary for the registration. Several authors proposed methods for an efficient selection of a subset of points (better than random) which does not degrade the quality of resulting registration. Goshtasby [15] used only points belonging to convex hull of the whole set. Convex hull is invariant to similarity transform and it forms sparse subset of the original data. However, there is a danger that convex hulls from reference and sensed images may differ from each other. Lavine [67] proposed to use points forming the minimum spanning trees of sets. Ehlers [39] merged points into "clumps" – large dense clusters.

Summary. To summarize, the feature-based methods are recommended to use if images contain enough distinctive and easily detectable objects. This is usually the case of applications in remote sensing and computer vision. Typical images contain a lot of details (towns, rivers, roads, forests, room facilities, etc). On the other hand, medical images do not have enough of such details and thus area-based methods are usually employed here. Sometimes, the lack of distinctive objects in medical images is solved by interactive selection by an expert or by introducing extrinsic features, rigidly positioned with respect to the patient (skin markers, screw markers, dental adapters etc.) [4]. An applicability of area-based and feature-based methods for images with various contrast and sharpness is analyzed in [68].

4 Feature matching

Detected features in the reference and sensed images can be matched by means of the image intensity values in their close neighborhoods, feature spatial distribution, or feature symbolic description. Some methods, while looking for the feature correspondence, simultaneously estimate the parameters of mapping functions and thus merge the second and third registration steps.

In the following paragraphs, the two major categories (area-based and feature-based methods, respectively) are retained and further classified into subcategories according to the basic ideas of the methods.

4.1 Area-based methods

Correlation-like methods. A classical representative of the area-based methods is normalized cross-correlation (CC) and its modifications [69]. This measure of similarity is computed for window pairs from the sensed and reference images and the maximum is searched. Window pairs for which the maximum is achieved are set as the corresponding ones. If the subpixel accuracy of the registration is demanded, the interpolation of the CC matrix values needs to be used. Although the cross-correlation based registration can exactly align mutually translated images only, it can be also successfully applied when slight rotation and scaling are present.

There are generalized versions of CC for geometrically more deformed images. They compute the cross-correlation for each assumed geometric transformation of the sensed image window [70] and are able to handle even more complicated geometric deformations than translation – usually similarity transform. Berthilsson [71] tried to register in this manner even affinely deformed images and Simper [72] proposed to use a divide and conquer system and a cross-correlation technique for registering images differing by perspective changes as well as changes due to the lens imperfections. The computational load, however, grows very fast with the increase of the transformation complexity.

For translated and rotated images, Goshtasby [12] proposed to calculate moment invariants from the circular-shaped windows and then to apply the cross-correlation criterion on the moment window representation. A two-stage algorithm was introduced to save the computational time. First, just rough moment description (the zero-order moment) is used and then, only at the positions that are likely to be correct, more detailed window representation (the second and third order moments) is computed to determine the best match. A similar idea was used earlier by Wong and Hall [73]. Along with moment-based window description, they applied hierarchical search strategy to match radar and optical images. Wider application of the moment based descriptors is, however, in features-based methods.

Similar to the cross-correlation methods is the sequential similarity detection algorithm (SSDA) [10]. It uses sequential search approach and computationally simpler distance measure than the cross-correlation. It accumulates the sum of absolute differences of the image intensity values (matrix l_1 norm) and applies the threshold criterion – if the accumulated sum exceeds the given threshold, the candidate pair of windows from the reference and sensed images is rejected and next pair is tested. The method is likely to be less accurate than the cross-correlation but it is faster.

Huttenlocher et al. [74] proposed a method working with another measure of image similarity. They registered binary images (the output of an edge detector), transformed by translation or translation plus rotation, by means of the Hausdorff distance (HD). They compared the HD based algorithm with the cross-correlation. Especially on images with perturbed pixel locations, which are problematic for cross-correlation, HD outperforms the cross-correlation.

Two main drawbacks of correlation-like methods are the flatness of the similarity measure maxima (due to the self-similarity of the images) and high computational complexity. The maximum can be sharpened by preprocessing or using edge or vector correlation. Pratt [9] applied, prior to the registration, image filtering to improve the CC performance on noisy or highly correlated images. Van Wie [75] and Anuta [76] employed the edge-based correlation, which is computed on the edges extracted from the images rather than on the original images themselves. Extension of this approach, called vector-based correlation, computes the similarity measures using various representations of the window (moment descriptors [12], for instance).

Finding the minimum of a dissimilarity or the maximum of a similarity functions is a multidimensional optimization problem, where the number of dimensions corresponds to the degrees of freedom of expected geometrical transformation. The simplest solution is an exhaustive search over the entire image. Although it is computationally demanding, it is often used if only translations are to be estimated. Althof et al. [11] proposed to decrease the necessary computational load by taking just a sparse regular grid of windows for which the CC matching is performed.

In case of transformations with more degrees of freedom, sophisticated optimization algorithms are required. An application of Gauss-Newton numerical minimization algorithm for minimizing the sum of squared differences is described in [77], where the projective geometric deformation was used. Levenberg-Marquardt optimization method was applied in [78] to minimize the variance in intensities of corresponding pixels. The images were registered by means of the projective transformation model plus lens distortion model. The combination of Levenberg-Marquardt method and the sum of squared differences metric is described in [79]. The Powell's multidimensional direction set method [80] is applied in Maes et al. [81]. Starink and Backer [82] tried to minimize a dissimilarity measure defined on point pairs by means of simulated annealing.

Vanderbrug and Rosenfeld concentrated in their work on the amount of computation needed for window pair testing. In [83], they used a subwindow first to find probable candidates of the corresponding window in the reference image and then the full-size window was applied. They discussed the appropriate choice of the subwindow size to minimize the expected computational cost. In [84] they proposed to use first both the sensed and reference images at coarser resolution and then, on locations with small error measure, to match higher resolution images. This technique is a simple example of pyramidal methods.

Despite the limitations mentioned above, the correlation-like registration is still often in use, particularly thanks to its easy hardware implementation, which makes it useful for real-time applications.

Fourier methods. If an acceleration of the computational speed is needed or if the images were acquired under varying conditions or they are corrupted by frequency-dependent noise, then Fourier methods are preferred rather than the correlation-like methods. They exploit the Fourier representation of the images in the frequency domain. The phase correlation method is based on the Fourier Shift Theorem [85] and was originally proposed for registration of translated images. It computes the cross-power spectrum of the sensed and reference images and looks for the location of a peak in its inverse. The method shows strong robustness against the correlated and frequency dependent noise and nonuniform, time varying illumination disturbances. The computational time savings are more significant if images, which are to be registered, are large.

De Castro and Morandi [86] introduced an extension of the phase correlation for additional rotation transform. If the change of image scale is present too, the images can be registered using the combination of polar-log mapping of the spectral magnitude (which corresponds to Fourier-Mellin transform) and the phase correlation [87], [88] or cepstrum filter [59]. Applications of the extended algorithm in remote sensing (SPOT images) and medical imaging (MR images) are described in [88]. Testing the accuracy of the method in simulated conditions (registration of deformed and noisy images of ocular fundus) was performed with satisfying results [89].

Another application exploiting the Fourier transform is described in [76]. The authors proposed to compute the correlation in frequency domain. This method can handle multimodal images when applied to the edge representations instead of the original graylevel images.

Mutual information methods. Mutual information (MI) methods are the last group of area-based methods to be reviewed here. They have appeared recently and represent the leading technique in multimodal registration. Registration of multimodal images is a difficult task, but often necessary to solve, especially in medical imaging. The comparison of anatomical and functional images of the patient body can lead to the diagnosis which would be impossible to gain otherwise. Remote sensing often makes use of the exploitation of more sensor types, too. Characteristics of such sensors are described, e.g., in [90].

Mutual information, originating in the information theory, is a measure of statistical dependency between two data sets (it measures the amount of information that one variable contains about the other) and it is particularly suitable for registration of images from different modalities. Often the speed up of the registration is achieved by exploiting the coarse-to-fine resolution strategy (pyramidal approach).

One of the first articles proposing this technique is [91]. The authors described the application of MI for the registration of magnetic resonance images as well as for 3-D object model matching to the real scene. MI was maximized using the gradient descent optimization method. Thévenaz and Unser [92], [93], [94] tried to combine various approaches to solve individual steps of MI registration. They employed Parzen window for the joint probability computation and Jeeves in [92] or Marquardt-Levenberg method in [93] to maximize the MI. To speed up the computation, they use spline pyramids in [94]. Ritter et al. [95] used hierarchical search strategy together with simulated annealing to find a maximum of the MI. Studholme et al. [96] compared three similar measures of information dependecy – joint entropy, MI and normalized MI (a new idea, claimed to outperform the others). They applied discrete histogram estimates of the joint probability distribution instead of Parzen window used in previous work. Maximization was achieved by using a multiresolution hill climbing algorithm. They registered MR-CT and MR-PET images of a human brain. Maes et al. [81] optimized the MI by means of Brent's method and Powell's multidimensional direction set method to register MR, CT and PET images of a human brain that differ by similarity transform. MI applied to breast MR images is described in [97]. The authors proposed to model the global deformation present between images by a combination of affine transformations and spline-based free form deformation. Likar and Pernus [98] studied the performance of different methods for joint probability estimation in registration of muscle fibre images. The comparison of the basic MI registration with the version employing the coarse-to-fine speed up was done in [99]. The comparison of MI to six other registration methods including normalized cross-correlation and gradient correlation is described in [100].

The above mentioned MI methods work with entire image data and directly with image intensities. Rangarajan et al. [101] applied MI on extracted features (points of the area borders), but this approach is still rare.

Summary. The area-based methods are applicable mainly to registration of images, misaligned by translation and perhaps small similarity or affine transformation. Registration of more complex geometrical transformations, particularly those with spatially local variations becomes computationally too demanding or even impossible. Moreover, these methods are in principle sensitive to unexpected gray-level modification like changes of illumination or blurring. Such situations require feature-based (symbolic) registration methods.

4.2 Feature-based methods

We now assume that two sets of features in the reference and sensed images represented by their control points (CP's) (points themselves, end points or centers of line features, centers of gravity of regions, etc.) have been detected. The aim is to find the pair-wise correspondence between them, using their spatial relations or various descriptors of features or of their neighborhoods.

Methods using spatial relations. Methods based primarily on the spatial relations among the features are usually applied if detected features are ambiguous or if their neighborhoods are locally distorted. The information about the distance between the CP's and their spatial distribution is exploited.

Goshtasby in [15] described a case of graph matching algorithm. He computed the number of features in the sensed image that, after the transformation, fall within a given range from the features in the reference image, for particular parameters of geometric transformation. The parameters with the highest score were then set as a valid estimate.

Clustering technique, presented by Stockman et al. [35], tries to match points connected by abstract edges, line segments. The assumed geometrical model is the similarity transform. For every pair of CP's from both the reference and the sensed images, the parameters of the transformation that maps the points on each other are computed and represented as a point in the space of transform parameters. The parameters of transformations that most closely map the highest number of features tend to form a cluster, while mismatches fill the parameter space randomly. The cluster is then detected and its centroid is assumed to represent the most probable matching parameters. Mapping function parameters are thus found simultaneously with the feature correspondence. Local errors do not influence globally the registration process.

Clustering technique was implemented, for example, in [13] and [102].

Barrow et al. [103] introduced the chamfer matching for image registration. Line features detected in images are matched by means of the minimalization of the generalized distance between them. Borgefors [104] proposed an improved version, where better measure of correspondence – sequential distance transform together with the root mean square average – was applied. The algorithm employs also the pyramidal speed-up.

Methods using invariant descriptors. As an alternative to the methods exploiting spatial relations, the correspondence of features can be estimated using their description, preferably invariant to expected image deformation. The description should fulfill several conditions. The most important ones are *invariance* (the descriptions of corresponding features from the reference and sensed image have to be the same), *uniqueness* (two different features should have different descriptions), *stability* (the description of a feature which is slightly deformed in an unknown manner should be the close to the description of the original feature), and *independence* (if the feature description is a vector, its elements should be functionally independent). However, usually not all these conditions have to (or can) be satisfied simultaneously and to find an appropriate trade-off is necessary.

Features from the sensed and reference images with the most similar invariant descriptions are paired as the corresponding ones. The choice of the type of the invariant description depends on the feature characteristics and the assumed geometric deformation of the images. While searching for the best matching feature pairs in the space of feature descriptors, minimum distance rule with thresholding is usually applied. If a more robust algorithm is needed, the matching likelihood coefficients [105], which can better handle questionable situations, can be an appropriate solution.

The simplest feature description is the image intensity function itself, limited to the close neighborhood of the feature [106], [59]. To estimate the feature correspondence, authors computed the cross-correlation on these neighborhoods. Zheng and Chellapa proposed to use the correlation coefficients. In [40], they assumed the similarity geometric deformation. Here, first the rotation between images was compensated by the estimation of the illuminant direction and then the coarse-to-fine correlation based registration was performed.

The following references are examples of the intuitive descriptions, which usually do not fulfill some of the mentioned criteria of invariant descriptors. Sester et. al. [18] proposed to use forests as the region features and describe them by elongation parameter, compactness, number of holes, and several characteristics of the minimum bounding rectangle. To register stars with a catalog, Murtagh [107] assigned to every point feature the description of the spatial distribution of other features lying around. Vujovic and Brzakovic in [30] represented every detected feature (elongated structure intersections) by the the signature in a form of the longest structure and angles between all other structures, participating in the intersection. Similarly, Zana [108] described each feature point by means of angles between adjacent lines.

Many authors used closed-boundary regions as the features. In principle, any invariant and discriminative shape descriptor can be employed in region matching. Peli [109] proposed simple and fast description by radial shape vector but the usage of this method is limited to star-shape regions only. A generalized shape description in a form of a binary matrix was proposed in [110], [111]. In [13], the shape matrices were used for registration of rotated and scaled satellite images. In [24], [25] a chain code representation of contours was proposed as the invariant description and a chain code correlation-like measure was used for finding the correspondence. Skea et al. [112] represented non-collinear triplets of CPs by the sphericity. Suk [113] proposed invariant shape description of the regions represented by polygons and further developed this approach in [114]

A large group of methods uses moment-based invariants for description of closed-boundary region features. Considering the most usual assumed deformations, Hu [115] introduced moment invariants to the similarity transform.

Flusser and Suk derived the affine transform invariants [116] and used them successfully when registering SPOT and Landsat images [14]. Holm [16] extracted closed boundary regions and proposed to represent them by their perimeter, area, compactness, moments, and moment invariants. Bhattacharya [47] suggested using complex moments. Brivio et al. [20] modeled shadow structures in mountain images by means of inertia ellipses. The ellipses are then described by their area, inclination of the main axis and ellipticity. All these attributes are functions of moments. Li et al. [24] used first two Hu's moments as preselectors for matching of closed contours. The candidate matches are then tested using the chain code representation of the contours. A similar method was described in [117], where moment invariants are used together with the chain codes. Sato and Cipolla [118] computed directly, without correspondence estimation, parameters of the present geometric deformations (an affine transform was expected) from circular moments of distribution of the orientation of line features. They combined moments and a scale-space representation of the images. Recently, Flusser and Suk [119] introduced a new class of moment invariants that are invariant to image blurring and demonstrated their performance in registering SPOT and AVHRR satellite images. Flusser et al. further developed this approach in [120] by introducing combined blur-rotation invariants. In [121] they generalized the previous invariants to register 3-D images.

Invariant combination of basic geometric properties of features can form geometrically oriented descriptors. Govindu et al. [26] represented extracted contours from possibly rotated images by the slopes of tangents in contour points. They did not look for contour correspondence, but only for the distributions of the proposed descriptors. They derived a similar type of descriptors for the affine transform, too. A detailed study of the practical aspects of the proposed method can be found in [122]. Wang and Chen [22] computed the histogram of line-length ratios and the histogram of angle differences of any two line segments in the reference and sensed images. They assumed the similarity transform. Griffin and Alexopoulos [123] used the ratio of the smallest enclosing circle radii, the difference of the locations of centroids of gravity, and sorting the neighbors lexicographically according to the angle with the x axis and the distance from the centroid. All these methods skip the step of finding the feature correspondence and establish directly the mapping function parameters.

Hsieh et al. [124] used the angle histogram computed on line feature points for the compensation of rotation difference. After removal of the rotation difference, the feature point correspondence is found by means of cross-correlation. They compared their rotation compensation with the one described in [40].

Shekhar et al. [125] combined different types of features and their descriptors. They decomposed the present geometric deformation into elementary steps and estimated corresponding parameters using a feature consensus – every type of feature descriptor votes for corresponding value of the parameter. The value of parameter which maximizes the number of votes over all descriptor types is then chosen.

Ventura et al. [126] described image features by various descriptors (ellipticity, angle, thinness, etc.) and represented relations among them by a multi-value logical tree (MVLT). Then they compared the MVLT's of the reference and sensed images to find the feature correspondence. MVLT's are applied also in [20], together with moment invariants.

Relaxation methods. A large group of the registration methods is based on the relaxation approach, as one of solutions to the consistent labeling problem (CLP): label each feature from the sensed image with the label of a feature from the reference image, so as it is consistent with the labeling given to the other feature pairs [127]. The process of recalculating the pair figures of merit, considering the match quality of the feature pairs and of matching their neighbors, is iteratively repeated until a stable situation is reached. The reference work was done by Ranade and Rosenfeld [128]. Here, the displacement of the feature sets transformed by a certain geometric transformation defines figures of merit of the feature pairs. This method can handle shifted images and tolerates local image distortions.

Wang et al. [48] extended the classical relaxation by inclusion of the description of corner features by their sharpness, contrast, and slope. This algorithm allows translation and rotation distortions of the images, but it is computationally demanding. Medioni and Nevatia [129] used line features and their descriptors (coordinates, orientation, average contrast). Cheng and Huang [130] proposed a star-based registration considering only each feature point along with all links to its neighbors. Ton and Jain [38] speeded up the algorithm by integrating the MERGESORT concept. Their method works with shifted and rotated images. Relaxation based method even for similarity transformed images was proposed, for example, by Cheng [131], Ogawa [132] and Li [133]. Different relaxation methods are compared in [134].

Another solution to the CLP problem and consequently to the image registration is backtracking, where consistent labeling is generated recursively. A registration method based on backtracking is described in [127].

Elastic deformation methods. Images with considerable local distortions or those deformed by a complicated transformation usually cannot be registered by parametric methods, where finding the geometric deformation is reduced to the search for the parameters of an *a priori* chosen transformation model. Feature descriptors which would be invariant to these complicated deformations are not known, so the correspondence of features would be difficult to establish. Thus, instead, elastic deformation methods are applied.

Here, images can be viewed as pieces of a rubber sheet, on which external forces stretching the image and internal forces defined by stiffness or smoothness constraints are applied to bring them into alignment with the minimal amount of bending and stretching. The feature matching and mapping function design steps of the registration are done simultaneously. The registration is achieved by locating the minimum energy state in an iterative fashion. A pyramidal approach is often applied. The definition of external forces can be derived from local optimization of the similarity function defined on intensity values or of the correspondence of boundary structures [135], among others. In [136], no external forces were used and prescribed displacements, derived from the correspondence of boundary structures, were incorporated to the elastic image deformation.

Miscellaneous. If the features be ordered using a linear ordering, dynamic programming can be used to found the feature correspondence. Dynamic programming prunes the search space of possible feature correspondences and solves the problem by means of solving subproblems. An example of such registration method is in [28], where the correspondence of coastal lines is looked for. The authors modified the classical dynamic programming algorithm to be able to cope with noisy satellite images with clouds partly covering the scene.

Vinod and Ghose [137] introduced a neural network based algorithm for solving the feature correspondence task. Their method can register images deformed by a translation, rotation, and noise. They compared the neural method with the relaxation approach. Gold et al. [138] proposed a neural network based method called soft-assign for the registration of 2-D and 3-D point sets differing by an affine transform.

Pyramids and wavelets. We conclude the discussion about the feature matching by mentioning some works trying to reduce the computational cost due to the large image size by means of pyramidal approach.

This coarse-to-fine hierarchical strategy applies usual registration methods, but it starts with the reference and sensed images on a coarse resolution (generated using Gaussian pyramids, simple averaging or wavelet transform coefficients, among others). Then, they gradually improve the estimates of the correspondence or parameters of the mapping function while going up to the finer resolutions. At every level, they considerably decrease the search space and thus save the necessary computational time. Another important advantage resides in the fact that the registration with respect to large-scale features is achieved first and then small corrections are made for finer details. On the other hand, this strategy fails if a false match is identified on a coarser level. To overcome this, a backtracking or consistency check should be incorporated into the algorithms.

Combining cross-correlation with the pyramidal approach exploiting a summing pyramid (the pixel value at a coarser level corresponds to the summation of the pixel values on the previous level), median pyramid, and averaging pyramid, was proposed in [139], [40] and [140] respectively. Wong and Hall [141] combined the SSDA method with pyramidal speed-up. Wang and Chen [22] extracted features (centroids of closed boundary regions) at every resolution level and found the parameters of geometric deformation from the histogram of angle differences and line-length ratios, as mentioned above. Thévenaz et al. applied a cubic spline based pyramid along with the minimization of the mean square intensity difference between the images [142] and the mutual information maximization [92], respectively. Sharma and Pavel [77] used the multiresolution Laplacian pyramid for the infrared and radar images registration. Kumar et al. [143] combined different types of pyramids (Laplacian, Gaussian) with different similarity measures (cross-correlation, sum of squared differences) to register aerial video sequences.

Recently, using a wavelet decomposition of the images was proposed for the pyramidal approach due to its inherent multiresolution character. Methods can differ in the type of applied wavelet and the set of wavelet coefficients used for finding the correspondence. Most frequently used methods decompose the image recursively into four sets of coefficients (LL, HL, LH, HH) by filtering the image successively with two filters, a low-pass filter L and a high-pass filter H, both working along the image rows and columns.

Turcajova and Kautsky [144] tested various orthogonal and biorthogonal wavelets (they used LL coefficients) together with cross-correlation on a regular grid of points to register affine transformed images. Spline biorthogonal wavelets and Haar wavelet outperformed others. Fonseca and Costa [44] detected the modulus maxima of LH and HL coefficients and looked for the maxima of the correlation coefficients, computed from LL coefficients in small surrounding of the detected maxima. Djamdji et al. [145] use just HH coefficients. Le Moigne [146] applied Daubechies wavelet to register Landsat images and AVHRR data. They extracted LH and HL frequency coefficients and found the correspondence by means of cross-correlation. The robustness of the registration by means of Daubechies and Haar wavelets was studied in [147].

5 Transform model estimation

After the feature correspondence has been established the mapping function is constructed. It should transform the sensed image to overlay it over the reference one. The correspondence of the control points from the sensed and reference images together with the fact that the corresponding CP pairs should be as close as possible after the sensed image transformation are employed in the mapping function design.

The task to be solved consists of choosing the type of the mapping function and its parameter estimation. The type of the mapping function should correspond to the assumed geometric deformation of the sensed image, to the method of image acquisition (scanner dependent distortions and errors) and to the required accuracy of the registration.

In special situations when the geometric deformation is partially known e.g., when there exists a model for the distortion caused by the acquisition device and/or the scene geometry, the pre-correction based on the inverse of the deformation can be performed (for example, in [27], [148] and [149] the authors model the Earth's shape and rotation, the satellite orbit and the scanning geometry of the sensor).

Models of mapping functions can be divided into two broad categories according to the amount of image data they use as the support. Global models use all CP's for estimating one set of the mapping function parameters valid for the entire image. On the other hand, local mapping functions treat the image as a composition of patches and function parameters depend on the location in the image. It leads to a tessellation of the image, usually a triangulation, and defining parameters of the mapping function for each patch separately.

From another point of view, mapping functions can be categorized according to the accuracy of overlaying the CP's used for computation of their parameters. Interpolating functions map the sensed image CP's on the reference image CP's exactly, whereas approximating functions try to find the best trade-off between the accuracy of the final mapping and other requirements imposed on the character of the mapping function. Since the CP coordinates are usually supposed not to be precise, the approximation model is more common.

One of the most frequently used global models uses bivariate polynomials of low degrees (similarity transform, affine transform or quadratic transform). An affine transform model assumes the distance of the camera to the scene is large in comparison to the size of the scanned area, the camera is perfect (a pin-hole camera), the scene is flat, and the present geometric distortion has no local factors. If the condition on the distance of the camera from the scene is not satisfied the perspective projection model can be appropriate. Combined slight violations of these conditions lead to the use of the second or the third order polynomial models. Higher order polynomials usually are not used in practical applications due to their wrong numerical properties.

In general, polynomials do not map the CP's onto their corresponding points exactly. Their parameters are computed by means of the least square method such that the polynomials minimize the sum of squared errors at the CP's. This approach proved to be very effective and accurate for satellite images, for instance.

However, a global polynomial mapping cannot properly handle images deformed locally. This happens, for instance, in medical imaging and in airborne imaging. The least square technique averages out the local geometric distortion equally over the entire image which is not desirable. Local areas of the image should be registered with the available information about local geometric distortion in mind.

Several authors have shown the superiority of the local or at least locally sensitive registration methods above the global ones in such situations (Goshtasby [150], Ehlers and Fogel [151], Wiemker [152], and Flusser [153], among others). The weighted least square and weighted mean methods [150] gain the ability to register images locally by introducing the slight variation to the original least square method. The local methods called piecewise linear mapping [154] and piecewise cubic mapping [155], together with the Akima's quintic approach [152], apply the combination of the CP-based image triangulation and of the collection of local mapping functions each valid within one triangle. These approaches belong to the group of the interpolating methods.

Radial basis functions are representatives of the group of global mapping methods but they are able to handle even locally varying geometric distortions. Resulting mapping function has a form of a linear combination of translated radially symmetric function plus a low-degree polynomial. Originally they were developed for the interpolation of irregular surfaces. Their name "radial" reflects an important property of the function value at each point – it depends just on the distance of the point from the control points, not on its particular position. Multiquadrics, reciprocal multiquadrics, Gaussians, Wendland's functions and thin-plate splines are several examples of the radial basis functions used in image registration.

An application of the multiquadrics in the airborne remote sensing, together with the comparison to the third order polynomial method, is described in [151]. Its comparison to the Akima's method is presented in [152]. A medical application of multiquadrics is shown in [156]. Wendland's functions applied in medical image registration appear in [157]. These functions have very small global influence and even significant local deformations can be well registered by this approach. This property is advantageous for registering medical images, where changes occur mainly locally.

The most often used representatives of the radial basis functions are the thin-plate splines (TPS). Although they had been used in mechanics and engineering for decades [158], they were introduced to image analysis community by Grimson [159] and Bookstein [160]. The TPS can be viewed as a very thin plate, which is fixed at the positions determined by the CP's in the reference image in the heights given by the x or y coordinates of the corresponding CP's in the sensed image [161]. The TPS minimizes a quadratic variation functional of potential energy that reflects the amount of function variation and which should be small for a good mapping function. The type of registration can be chosen – both exact interpolation [161] and approximation [162] are possible. This method gives good results but the computations can be very time consuming.

Several successful attempts to decrease the computational time to the prejudice of the registration accuracy have been described. Adaptive approximation of the TPS by simpler functions [153], expansion of the TPS into power series [163] and tabulation of the TPS [164] are examples of efficient numeric algorithms. Barrodale et al. [165] paid attention to fast and robust calculation of the TPS coefficients.

Numerous papers have dealt with comparison of the performance of thin-plate splines and other mapping functions. In [161], [152] and [166] they are compared with polynomials and multiquadrics when registering aerial images. The comparison of the TPS with the Gaussian radial basis functions and with multiquadrics considering their local properties was done in [167]. It was concluded that the TPS have favorable properties when used as mapping functions for image registration, while the other radial basis functions are more convenient for image warping and morphing. Many image analysis applications outside the area of image registration take advantage of the TPS: animation of facial expressions [167], shape recovery from stereo and/or from depth map [168], [159], interpolating incomplete surfaces [169], image sharpening and scaling [170] and noise filtering via image smoothing [171], [172].

The TPS are not the only representatives of spline family used for the mapping function design. A linear combination of translated cubic B-splines was used for the registration of echo planar images [173]. Another type of spline-based function, elastic body spline (EBS), was proposed in [174]. It evolved from the description of the equilibrium displacements of homogeneous isotropic elastic material subjected to a load. Body tissues depicted in the image data to be registered often have properties of elastic material. The EBS was used for the registration of 3-D MRI images of breasts. The authors claimed the EBS had outperformed the TPS in their experiments.

6 Image resampling and transformation

The mapping functions constructed during the previous step are used to transform the sensed image and thus to register the images. The transformation can be realized in a forward or backward manner. Each pixel from the sensed image can be directly transformed using the estimated mapping functions. This approach, called a forward method, is complicated to implement, as it can produce holes and/or overlaps in the output image (due to the discretization and rounding). Hence, the backward approach is usually chosen. The registered image data from the sensed image are determined using the coordinates of the target pixel (the same coordinate system as of the reference image) and the inverse of the estimated mapping function. The image interpolation takes place in the sensed image on a regular grid. In this way neither holes nor overlaps can occur. The interpolation itself is usually realized via convolution of the image with an interpolation kernel. An optimal interpolant – the *sinc* function – is hard to implement in practice because of its unlimited support. Thus, many simpler interpolants of bounded support have been investigated in the literature.

The nearest neighbor function, the billinear and bicubic functions, quadratic splines [175], [176], cubic B-splines [177], Catmull- Rom cardinal splines [178], [142], Gaussians [179], and truncated sinc functions [180] belong to the most commonly used interpolants. Meijering et al. [181] investigated higher-order polynomial kernels (quintic and septic). However, their experiments showed only marginal improvement in comparison with cubic interpolation at an highly increased computational cost. Unser et al. [182] proposed the scheme where only 1-D interpolants are employed to resample 2-D data and showed it to be superior in terms of speed.

Several survey papers on resampling techniques have been published in the last years. A detailed investigation and comparison of the above methods was carried out in [183] for 2-D images and in [184] for 3-D data. Thevenaz et al. [180] paid attention to the elimination of undesired interpolation artifacts. Lehman et al. [185] published a survey article covering main interpolation methods (various versions of *sinc* functions, nearest neighbor, linear, quadratic, cubic, cubic B-spline, Lagrange and Gaussian kernels) with the emphasis on medical imaging applications. They compared them using the spatial and Fourier analysis and tested the computational complexity as well as interpolation errors. Most recently, Thevenaz et al. [186] have proposed a different approach to image resampling. Unlike the other methods, their resampling functions do not necessarily interpolate the image graylevels. They rather interpolate values calculated as certain functions of the graylevels. The authors have demonstrated this approach outperforms traditional interpolation techniques.

Even though the billinear interpolation is outperformed by other methods in terms of accuracy and visual appearance of the transformed image, it is still probably the best trade-off between accuracy and computational complexity and thus the most commonly used approach. Cubic interpolation is recommended when the geometric transformation involves a significant enlargement of the sensed image.

7 Current trends and outlook for the future

Image registration is one of the most important tasks when integrating and analyzing information from various sources. It is a key stage in image fusion, change detection and in building image information systems, among others. This paper gives a survey of the classical and up-to-date registration methods, classifying them according to their nature as well as according to the four major registration steps. Although a lot of work has been done, automatic image registration still remains an open problem. Registration of images with complex nonlinear and local distortions, multimodal registration and registration of N-D images (where $N > 2$) belong to the most challenging tasks at this moment.

When registering images with nonlinear, locally dependent geometric distortions, we are faced with two basic problems - how to match the control points and what mapping functions to use for registration. While the second one can be solved at least on theoretical level by using appropriate radial basis functions, the first problem is generally unsolvable due to its nature. Since the between-image deformations can be arbitrary, we cannot use any automatic matching method. Another conceptual question here is how can we distinguish between image deformations and real changes of the scene.

In multimodal registration, mutual information technique has become a standard reference, mainly in medical imaging. However, being an area-based technique, the mutual information has principal limitations. To overcome them, some authors combined the mutual information with other, preferably feature-based, methods to gain higher robustness and reliability. To speed up the computation, they often employed pyramidal image representation along with fast optimization algorithms. Unfortunately, when the images have significant rotation and/or scaling differences, these methods either fail or become extremely time expensive. The future development on this field should pay more attention to the feature-based methods, where appropriate invariant and modality-insensitive features can provide good platform for the registration.

Besides, we trust that new application-specific methods utilizing particular sensor characteristics appear soon in remote sensing.

The major difficulty of N-D image registration resides in its computational complexity. Although the speed of computers has been growing, the need to decrease the computational time of methods persists. The complexity of methods as well as the size of data still grows (the higher resolution, higher dimensionality, larger size of scanned areas). Moreover, the demand for higher robustness and accuracy of the registration usually enforces solutions utilizing the iterations or backtracking, which also produces increase of computational complexity of the method.

In the future, the idea of an ultimate registration method, able to recognize the type of given task and to decide by itself about the most appropriate solution, can motivate the development of expert systems. They will be based on the combination of various approaches, looking for consensus of particular results.

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