Graph flip processes related to dynamical systems

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Abstract We define a discrete-time "graph flip process" starting with an arbitrary n-vertex graph G_0. At each step, G_i is obtained from G_{i-1} by modifying a subgraph induced by a uniformly chosen set of k vertices (with k fixed) according to a predefined rule that depends only on the structure of the induced subgraph. For example, the rule might be "if the subgraph is a clique, remove its edges" or "replace the subgraph with a random graph regardless of its structure".

Fixing a constant $T > 0$, we analyze the concentration of trajectories $(G_0, \dots, G_{Tn^2})$ as $n \to \infty$ using the theory of dense graph limits. We show that the mean trajectory can be interpreted as a solution to a dynamical system on the space of graphons, which are functions in $L^\infty([0,1]^2)$.

We also describe properties of several specific processes, including the Erdős–Rényi process and the triangle removal process, and pose a couple of open questions related to these processes.

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References

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