On the contact process on dynamical random graphs with degree dependent dynamics

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Abstract Recently, there has been increasing interest in interacting particle systems on evolving random graphs, respectively in time evolving random environments. In this talk we present results on the contact process in an evolving edge random environment on infinite (random) graphs. The classical contact process models the spread of an infection in a structured population. The structure is given by a graph and the infection is passed on along the edges with rate λ while recovery from the infection happens spontaneously with rate 1. In an edge random environment the edges of the underlying (random) graph may be be dynamically opened and closed to infection.

We first give an overview over recent results. Then, we in particular consider (infinite) Bienaymé-Galton-Watson (BGW) trees as the underlying random graph. Here, we focus on an edge random environment that is given by a dynamical percolation whose opening and closing rates and probabilities are degree dependent. This means that any edges between two vertices x and y with degrees d_x and d_y is independently updated with rate $v(d_x, d_y)$ and subsequently again declared open (or otherwise closed) with probability $p(d_x, d_y)$. Our results concern the impact of v and p on the critical infection rate for weak (global) and strong (local) survival of the infection. Specifically, we establish conditions under which the contact process undergoes a phase transition.

For a general connected locally finite graph we provide sufficient conditions for the critical infection rate to be strictly positive. Furthermore, in the setting of BGW trees, we provided conditions on the offspring distribution as well as on v and p so that the process survives strongly with positive probability for all positive values of the infection rate.

In particular, if the offspring distribution follows a power law (or has a stretched exponential tail) and the connection probability is given by a product kernel, i.e., $p(d_x, d_y) = (d_x d_y)^{-\alpha}$ for a positive α, (or a maximum kernel) and the update speed exhibits polynomial behaviour, we provide a quite complete characterisation of the phase transition.

This talk is based on joint work with Natalia Cardona-Tobón, Marcel Ortgiese and Marco Seiler.